1.1 ALGEBRAIC EXPRESSION

Definition of an algebraic expression.

- An algebraic expression is an expression that contains one or more numbers, one or more variables (like x or y), and one or more arithmetic operations (like add, substract, multiply and divide).

- Example of algebraic expression.

$$5x^2 - 6yx + 7$$

- 5 and 6 are coefficients.

  coefficient is a number used to multiply a variable

- x and y are variables.

  Variable is a symbol, usually a letter, that represent one or more numbers

- 2 is a exponent (power).

  Exponent of a number shows how many times the number is to be used in a multiplication

Simplify Algebraic Expression.

- Basic properties of algebra are to rewrite an algebraic expression in a simpler form.
- To **simplify** an algebraic expression generally means to remove symbols of grouping such as parentheses or brackets and combine like terms.
- Two or more terms of an algebraic expression can be combined only if they are like terms.

Like Terms	Not Like Terms
$3x\ and - 2x$	$7x\ and\ 8y$
$-3x^2\ and\ 9x^2$	$5y^2 and\ 2y$
$-7x^2y^3\ and\ 15y^3x^2$	$xy^2\ and\ x^2y$

1 Example

Simplify each expression.

(a) $9y - 6 - 3(2 - 5y) = 9y - 6 - 3(2) - 3(-5y)$ Expand the brackets.

$= 9y - 6 - 6 + 15y$

$= 24y - 12$ Combine like terms.

(b) $5(2x^2 - 6x) - 3(4x^2 - 9) = 10x^2 - 30x - 12x^2 + 27$

$= -2x^2 - 30x + 27$ Combine like terms

(c) $2a^2 + 3a - 5a^2 - a = (2a^2 - 5a^2) + (3a - a)$ Group like terms

$= -3a^2 + 2a$

(d) $5y - 2y[3 + 2(y - 7)] = 5y - 2y(3 + 2y - 14)$

$= 5y - 2y(-11 + 2y)$

$= 5y + 22y - 4y^2$

$= 27y - 4y^2$

(e) $3xy^2 + 4x^2y^2 - 2xy^2 - (xy)^2 = 3xy^2 + 4x^2y^2 - 2xy^2 - x^2y^2$

$= (3xy^2 - 2xy^2) + (4x^2y^2 - x^2y^2)$

$= xy^2 + 3x^2y^2$

(f) $3x(x-3)(x+7) - 5x^3 + 7 = 3x[x(x) + x(7) - 3(x) - 21] - 5x^3 + 7$

$= 3x[x^2 + 7x - 3x - 21] - 5x^3 + 7$

$= 3x[x^2 + 4x - 21] - 5x^3 + 7$

$= 3x^3 + 12x^2 - 63x - 5x^3 + 7$

$= -2x^3 + 12x^2 - 63x + 7$

2 Example

Express the following algebraic fractions as the single friction in its simplest form.

a) $\dfrac{m-2}{2m^2}-\dfrac{3}{4m}$

$=\dfrac{m-2}{2m^2}\times\left(\dfrac{2}{2}\right)-\dfrac{3}{4m}\times\left(\dfrac{m}{m}\right)$ equal the denominator

$=\dfrac{2(m-2)-3m}{4m^2}$ simplify the numerator

$=\dfrac{2m-4-3m}{4m^2}=\dfrac{-m-4}{4m^2}$

b) $\dfrac{p-1}{p^2}-\dfrac{q-1}{pq}$

$=\dfrac{p-1}{p^2}\times\left(\dfrac{q}{q}\right)-\dfrac{q-1}{pq}\times\left(\dfrac{p}{p}\right)$ equal the denominator

$=\dfrac{q(p-1)-(q-1)p}{p^2q}$ simplify the numerator

$=\dfrac{pq-q-pq+p}{p^2q}=\dfrac{p-q}{p^2q}$

c) $\dfrac{7}{6a}-\dfrac{5-7a}{4a^2}$

$=\dfrac{7}{6a}\times\left(\dfrac{2a}{2a}\right)-\dfrac{5-7a}{4a^2}\times\left(\dfrac{3}{3}\right)$

$=\dfrac{14a}{12a^2}-\dfrac{15-21a}{12a^2}$

$=\dfrac{14a-15+21a}{12a^2}$

$=\dfrac{35a-15}{12a^2}=\dfrac{5(7a-3)}{12a^2}$

3 Example

Solve the following equation

(a) $4 - x = 5$

$4 - x - 4 = 5 - 4$ Substract -4 both sides

$-x \times (-1) = 1 \times (-1)$ Multiply by -1

$x = -1$

(b) $-\frac{5}{3}x = 10$

$-\frac{5}{3}x \times \left(-\frac{3}{5}\right) = 10 \times \left(-\frac{3}{5}\right)$ Multiply by $\left(-\frac{3}{5}\right)$

$x = -6$

(c) $5x - 3 = 32$

$5x - 3 + 3 = 32 + 3$ Add 3

$\frac{5x}{5} = \frac{35}{5}$ Divide by 5

$x = 5$

(d) $13 - 5x = 8(x - 10)$ Expand the brackets

$13 - 5x = 8x - 80$

$-5x - 8x = -80 - 13$ Combine like terms

$-13x = -93$

$-\frac{13x}{-13} = -\frac{93}{-13}$ Divide by -13

$x = \frac{93}{13}$

(e) $3x(2 + x) = x(3x - 2) - 24$ Expand the brackets

$6x + 3x^2 = 3x^2 - 2x - 24$

$6x + 3x^2 - 3x^2 + 2x = -24$ Combine like terms

$\frac{8x}{8} = \frac{-24}{8}$

$x = -3$

1.2 QUADRATIC EQUATION

Definition of a quadratic equation.

- An equation with one variable only. (like x or y)
- The highest power of variable is 2.
- An equation has equal sign (=).

General form of a quadratic equation.

- $ax^2 + bx + c = 0$ where *a, b* and *c* are constant and *a* ≠ 0.

4 Example

Express the following quadratic equations in the general form $ax^2 + bx + c = 0$ and state the values of *a, b* and *c*.

(a) $x^2 = 6x - 10$ Compare with the general form $ax^2 + bx + c = 0$.

$x^2 - 6x + 10 = 0$ thus $a = 1, b = -3, c = 7$

(b) $(3x - 5)(x - 3) = 2$

$3x(x) + 3x(-3) - 5(x) - 5(-3) = 2$ Compare with the general form $ax^2 + bx + c = 0$

$3x^2 - 9x - 5x + 13 = 0$

$3x^2 - 14x + 13 = 0$ thus $a = 3, b = -14, c = 13$

Solving Quadratic Equation.

- Solving a quadratic equation $ax^2 + bx + c = 0$ means finding the roots of the equation.
- The roots of a quadratic equation can be solved by factorization, completing the square and using the formula.

Method A. Factorization

5 Example

Solve the following quadratic equations by factorization.

(a) $x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x + 5 = 0$ or $x - 2 = 0$

$x = -5$ $x = 2$

Therefore $x = -5$ and $x = 2$ are roots of $x^2 + 3x - 10 = 0$.

factorise

	x	5	$5x$
$(\times)$	x	-2	$-2x$ $(+)$
	x^2	-10	$3x$

(b) $2x^2 = 11x - 12$

$2x^2 - 11x + 12 = 0$

$(2x-3)(x-4) = 0$

$2x - 3 = 0$ or $x - 4 = 0$

$x = \frac{3}{2}$ $x = 4$

Therefore $x = \frac{3}{2}$ and $x = 4$ are roots of $2x^2 = 11x - 12$

factorise

	$2x$	-3	$-3x$
$(\times)$	x	-4	$-8x$ $(+)$
	x^2	12	$-11x$

(c) $x(x+4) = -3$

$x^2 + 4x + 3 = 0$

$(x+3)\ (x+1) = 0$

$x + 3 = 0$ or $x + 1 = 0$

$x = -3$ $x = -1$

Therefore $x = -3$ and $x = -1$ are roots of $x(x+4) = -3$

factorise

	x	3	$3x$
$(\times)$	x	1	$8x$ $(+)$
	x^2	3	$11x$

Note : Check the answers using calculator.

- Press [MODE] three times. Than press [1] for [EQN].
- Press [Degree] and substitute value of a, b and c. finally equal [=] sign.

Method B. Completing the square

6 Example

Solve the following quadratic equations by completing the square.

a) $x^2 + 4x - 12 = 0$

$$x^2 + 4x = 12$$

$$x^2 + 4x + \left(\frac{4}{2}\right)^2 = 12 + \left(\frac{4}{2}\right)^2$$

> Add $\left(\frac{\text{Coefficient of } x}{2}\right)^2$ on both sides of the equation

$$x^2 + 4x + (2)^2 = 12 + (2)^2$$

$$(x+2)^2 = 16$$

> Use $(x+a)^2 = x^2 + 2ax + a^2$

$$(x+2) = \pm\sqrt{16}$$

$$x = -2 \pm 4$$

$x = -2 + 4$ or $x = -2 - 4$

$x = 2$ $\qquad$ $x = -6$

Therefore $x = 2$ and $x = -6$ are roots of $x^2 + 4x - 12 = 0$

b) $3x^2 - 14x + 10 = 0$

$$\frac{3}{3}x^2 - \frac{14}{3}x = -\frac{10}{3}$$

$$x^2 - \frac{14}{3}x = -\frac{10}{3}$$

$$x^2 - \frac{14}{3}x + \left(\frac{-14/3}{2}\right)^2 = -\frac{10}{3} + \left(\frac{-14/3}{2}\right)^2$$

$$x^2 - \frac{14}{3}x + \left(-\frac{7}{3}\right)^2 = -\frac{10}{3} + \left(-\frac{7}{3}\right)^2$$

$$\left(x - \frac{7}{3}\right)^2 = \frac{19}{9}$$

$$\left(x - \frac{7}{3}\right) = \pm\sqrt{\frac{19}{9}}$$

$$x = \frac{7}{3} \pm \sqrt{\frac{19}{9}}$$

$x = \frac{7}{3} + \sqrt{\frac{19}{9}} = 3.79$ or $x = \frac{7}{3} - \sqrt{\frac{19}{9}} = 0.88$

Method C. Quadratic Formula

There is a quadratic formula, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

7 Example

By using the quadratic formula method, solve the following quadratic equations.

a) $m^2 - 5m - 14 = 0$

compare with $ax^2 + bx + c = 0$

$a = 1,\ b = -5\ and\ b = -14$

substitution in a formula

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 + 56}}{2}$$

$$x = \frac{5 \pm \sqrt{81}}{2}$$

$$x = \frac{5 \pm 9}{2}$$

Hence, the solutions are

$x = \frac{5+9}{2} = 7$ and $x = \frac{5-9}{2} = -2$

b) $21x^2 = 13x + 20$

$21x^2 - 13x - 20 = 0$

$a = 21, b = -13\ and\ c = -20$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(21)(-20)}}{2(21)}$$

$$x = \frac{13 \pm 43}{42}$$

hence, the solutions are

$x = \frac{13+43}{42} = \frac{4}{3}$ *and* $x = \frac{13-43}{42} = -\frac{5}{7}$

1.3 PARTIAL FRACTION

- An algebraic fraction is a fraction in which the numerator and denominator are both polynomial expressions.

- A polynomial expression is one where every term is a multiple of power of x, such as

$$3x^3 - 7x^2 + 3x + 10$$

- The degree of a polynomial is the power of the highest term in x. Therefore the degree is 3.

- Definition of **<u>proper fraction</u>**.
  - The degree of numerator is less than the degree of the denominator of fraction.

  For example $\quad \dfrac{x^2-4}{x^3+2x} \qquad \dfrac{x}{x^2-3}$

- Definition of **<u>improper fraction</u>**.
  - The degree of numerator is greater than or equal to the degree of the denominator of fraction.

  For example $\quad \dfrac{x^4+7x}{x^2+2} \qquad \dfrac{x^2}{x^2-3}$

- Definition of **<u>partial fraction</u>**.
  - The proper fraction is decomposed into a sum of two or more simpler proper fraction.

  For example $\quad \dfrac{x-4}{x(x+4)} = \dfrac{-1}{x} + \dfrac{2}{x+4}$

- Convert improper fraction to mixed number by using long division.

8 Example

Convert the following improper fractions to mixed numbers.

a) $\frac{x^3}{x^3-3}$

First, do the long division to find the regular polynomial part and the remainder

$$\begin{array}{r l} & \quad 1 \\ x^3-3 \,\big) & \; x^3 \\ (-) & \; \underline{x^3-3} \\ & \quad 3 \end{array}$$

The polynomial on top is "1" and the remainder is 3. Since you're dividing by "x - 3", the fractional part will be "(1)/(x - 3)".

$$\frac{x^3}{x^3-3} = 1+\frac{3}{x^3-3}$$ ➡ Mixed number

b) $\frac{x^2+4x+3}{x+2}$

$$\begin{array}{r l} & \qquad\quad x+2 \\ x+2 \,\big) & \; x^2+4x+3 \\ (-) & \; \underline{x^2+2x} \\ & \qquad 2x+3 \\ (-) & \qquad \underline{2x+4} \\ & \qquad\quad -1 \end{array}$$

$$\frac{x^2+4x+3}{x+2} = x+2+\frac{-1}{x+2}$$

$$= x+2-\frac{1}{x+2}$$

- Rules to determine the terms in the decomposition.
  - For a Linear term of denominator, such as $ax+b$ we get contribution of $\boxed{\dfrac{A}{ax+b}}$.
  - For a repeated linear term, such as $(ax+b)^3$, we get contribution of

    $$\boxed{\frac{A}{ax+b}+\frac{B}{(ax+b)^2}+\frac{C}{(ax+b)^3}}$$
  - For a quadratic term, such as ax^2+bx+c, we get contribution of

    $$\boxed{\frac{Ax+b}{ax^2+bx+c}}$$

<u>A) Partial Fraction using Proper Fraction with Linear factor</u>

9 Example

Express the following as a sum of partial fractions

a) $\dfrac{2x-1}{(x+2)(x-3)}$

Since the denominator in form of linear factor, than

$$\frac{2x-1}{(x+2)(x-3)}=\frac{A}{x+2}+\frac{B}{x-3}$$

multiply both sides with common denominator $(x+2)(x-3)$,

$$(x+2)(x-3)\times\frac{2x-1}{(x+2)(x-3)}=\left(\frac{A}{x+2}+\frac{B}{x-3}\right)\times(x+2)(x-3)$$

$$\cancel{(x+2)(x-3)}\times\frac{2x-1}{\cancel{(x+2)(x-3)}}=\frac{A}{\cancel{x+2}}\times\cancel{(x+2)}(x-3)+\frac{B}{\cancel{x-3}}\times(x+2)\cancel{(x-3)}$$

$$2x-1=A(x-3)+B(x+2)$$

Using substitution method,

let $x=3$, $\quad 2(3)-1=A(3-3)+B(3+2)$

$$5=0+5B$$

$$1=B$$

let $x=-2$ $\qquad 2(-2)-1 = A(-2-3)+B(-2+2)$

$$-5 = -5A+0$$

$$1 = A$$

Therefore, $\dfrac{2x-1}{(x+2)(x-3)} = \dfrac{1}{x+2}+\dfrac{1}{x-3}$ — Partial Fraction

10 Example

b) $\dfrac{x-4}{x(x+4)}$

Since the denominator in form of linear factor, than

$$\frac{x-4}{x(x+4)} = \frac{A}{x}+\frac{B}{x+4}$$

multiply both sides with common denominator $x(x+4)$,

$$x(x+4)\times\frac{x-4}{x(x+4)} = \left(\frac{A}{x}+\frac{B}{x+4}\right)\times x(x+4)$$

$$\cancel{x(x+4)}\times\frac{x-4}{\cancel{x(x+4)}} = \frac{A}{\cancel{x}}\times \cancel{x}(x+4)+\frac{B}{\cancel{x+4}}\times x\cancel{(x+4)}$$

$$x-4 = A(x+4)+Bx$$

Using substitution method,

let $x=-4$, $\qquad -4-4 = A(-4+4)+B(-4)$

$$-8 = 0-4B$$

$$2 = B$$

let $x=0$ $\qquad 0-4 = A(0+4)+B(0)$

$$-4 = 4A+0$$

$$-1 = A$$

Therefore, $\dfrac{x-4}{x(x+4)} = \dfrac{-1}{x}+\dfrac{2}{x+4}$

B) Partial Fraction using Proper Fraction with Repeated Linear Factors

11 Example

Express the following as a sum of partial fractions.

a) $\dfrac{3x+1}{(x-1)^2(x+2)}$

Since the denominator in form of repeated linear factors, than

$$\frac{3x+1}{(x-1)^2(x+2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+2)}$$

Multiply both sides with common denominator $(x-1)^2(x+2)$,

$$(x-1)^2(x+2)\times\frac{3x+1}{(x-1)^2(x+2)}=\left(\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+2)}\right)\times(x-1)^2(x+2)$$

$$3x+1=A(x-1)(x+2)+B(x+2)+C(x-1)^2$$

Using substitution method,

let $x=1$, $\quad 3(1)+1=A(1-1)(1+2)+B(1+2)+C(1-1)^2$

$$4=0+3B+0$$

$$\frac{4}{3}=B$$

let $x=-2$, $\quad 3(-2)+1=A(-2-1)(-2+2)+B(-2+2)+C(-2-1)^2$

$$-5=0+0+9C$$

$$-\frac{5}{9}=C$$

Using equating coefficients method to find value of A,

$$3x+1=A(x^2+x-2)+Bx+2B+Cx^2-2Cx+C$$

$$3x+1=Ax^2+Ax-2A+Bx+2B+Cx^2-2Cx+C$$

$$3x+1=(A+C)x^2+(A+B-2C)x-(2A-2B-C)$$

Compare both sides,

$A+C=0$, $\quad A=-C$, $\quad$ *So* $A=-\dfrac{5}{9}$

Therefore, $\dfrac{3x+1}{(x-1)^2(x+2)}=\dfrac{5}{9(x-1)}+\dfrac{4}{3(x-1)^2}-\dfrac{5}{9(x+2)}$

12 Example

b) $\dfrac{5x^2+17x+15}{(x+1)(x+2)^2}$

Since the denominator in form of repeated linear factors, than

$$\frac{5x^2+17x+15}{(x+1)(x+2)^2}=\frac{A}{(x+1)}+\frac{B}{(x+2)}+\frac{C}{(x+2)^2}$$

Multiply both sides with common denominator $(x-1)^2(x+2)$,

$$(x+1)(x+2)^2\times\frac{5x^2+17x+15}{(x+1)(x+2)^2}=\left(\frac{A}{(x+1)}+\frac{B}{(x+2)}+\frac{C}{(x+2)^2}\right)\times(x+1)(x+2)^2$$

$$5x^2+17x+15=A(x+2)^2+B(x+1)(x+2)+C(x+1)$$

Using substitution method,

let $x=-2$, $\quad 5(-2)^2+17(-2)+15=A(-2+2)^2+B(-2+1)(-2+2)+C(-2+1)$

$$1=A(0)+B(0)+C(-1)$$

$$-1=C$$

let $x=-1$, $\quad 5(-1)^2+17(-1)+15=A(-1+2)^2+B(-1+1)(-1+2)+C(-1+1)$

$$3=A(1)^2+B(0)+C(0)$$

$$3=A$$

Using equating coefficients method to find value of A,

$$5x^2+17x+15=A(x^2+4x+4)+B(x^2+3x+2)+C(x+1)$$

$$5x^2+17x+15=Ax^2+4Ax+4A+Bx^2+3Bx+2B+Cx+C$$

$$5x^2+17x+15=(A+B)x^2+(4A+3B+C)x+(4A+2B+C)$$

Compare both sides,

$A+B=5,\qquad A=5-B,\qquad So\quad \mathrm{B}=2$

Therefore, $\dfrac{5x^2+17x+15}{(x+1)(x+2)^2}=\dfrac{3}{(x+1)}+\dfrac{2}{(x+2)}-\dfrac{1}{(x+2)^2}$

C) Partial Fraction using Proper Fraction with Quadratic Factors

13 Example

Express the following as a sum of partial fractions.

a) $\dfrac{13}{(2x+3)(x^2+1)}$

Since the denominator in form of quadratic factors, than

$$\frac{13}{(2x+3)(x^2+1)} = \frac{A}{(2x+3)} + \frac{Bx+C}{(x^2+1)}$$

Multiply both sides with common denominator $(2x+3)(x^2+1)$,

$$(2x+3)(x^2+1) \times \frac{13}{(2x+3)(x^2+1)} = \left(\frac{A}{(2x+3)} + \frac{Bx+C}{(x^2+1)}\right) \times (2x+3)(x^2+1)$$

$$13 = A(x^2+1) + (Bx+C)(2x+3)$$

Using substitution method,

$let\ x = -\dfrac{3}{2},$ $\qquad 13 = A\left(\left(-\dfrac{3}{2}\right)^2 + 1\right) + \left(B\left(-\dfrac{3}{2}\right) + C\right)\left(2\left(-\dfrac{3}{2}\right) + 3\right)$

$$13 = \frac{13}{4}A + \left(-\frac{3}{2}B + C\right)(0)$$

$$4 = A$$

Using equating coefficients method to find value of A,

$$13 = A(x^2+1) + (Bx+C)(2x+3)$$

$$13 = Ax^2 + A + 2Bx^2 + 3Bx + 2Cx + 3C$$

$$13 = (A+2B)x^2 + (3B+2C)x + A + 3C$$

Compare both sides,

$A + 2B = 0,\qquad \mathrm{B} = -\dfrac{A}{2},\qquad So\quad \mathrm{B} = -\dfrac{4}{2} = -2$

$A + 3C = 13,\qquad 3\mathrm{C} = 13 - A,\qquad So\quad \mathrm{C} = \dfrac{9}{3} = 3$

Therefore, $\dfrac{13}{(2x+3)(x^2+1)} = \dfrac{4}{(2x+3)} + \dfrac{3-2x}{(x^2+1)}$

14 Example

b) $\dfrac{5x}{(x^2+x+1)(x-2)}$

Since the denominator in form of quadratic factors, than

$$\frac{5x}{(x^2+x+1)(x-2)} = \frac{Ax+B}{(x^2+x+1)} + \frac{C}{(x-2)}$$

Multiply both sides with common denominator $(x^2+x+1)(x-2)$,

$$(x^2+x+1)(x-2)\times\frac{5x}{(x^2+x+1)(x-2)} = \left(\frac{Ax+B}{(x^2+x+1)} + \frac{C}{(x-2)}\right)\times(x^2+x+1)(x-2)$$

$$5x = (Ax+B)(x-2) + C(x^2+x+1)$$

Using substitution method,

let $x = 2,$ $\quad 5(2) = (2A+B)(2-2) + C(2^2+2+1)$

$$10 = (2A+B)(0) + 7C$$

$$\frac{10}{7} = C$$

Using equating coefficients method to find value of A,

$$5x = (Ax+B)(x-2) + C(x^2+x+1)$$

$$5x = Ax^2 - 2Ax + Bx - 2B + Cx^2 + Cx + C$$

$$5x = (A+C)x^2 + (-2A+B+C)x + (-2B+C)$$

Compare both sides,

$A+C=0,$ $\quad$ $\mathrm{A} = -C,$ $\quad$ *So* $\mathrm{A} = -\dfrac{10}{7}$

$-2B+C=0,$ $\quad$ $\mathrm{B} = \dfrac{C}{2},$ $\quad$ *So* $\mathrm{B} = \dfrac{5}{7}$

Therefore, $\dfrac{5x}{(x^2+x+1)(x-2)} = \dfrac{-\frac{10}{7}x+\frac{5}{7}}{(x^2+x+1)} + \dfrac{\frac{10}{7}}{(x-2)} = \dfrac{-10x+5}{7(x^2+x+1)} + \dfrac{10}{7(x-2)}$

PRACTICE

1. Simplify the following algebraic expressions.
   a) $10x-3y+15x+20y$
   b) $8b^2+\frac{5}{2}b-\frac{3}{2}b^2+10b$
   c) $-5k^3+3k-6k^2+8k^3+k-4$
   d) $3\mathrm{p}^2q^2-4pq-q(5p)+2(pq)^2$
   e) $3(\mathrm{y}^2+1)+y^2-6$
   f) $6(\mathrm{m}^2-m-2)-2(3m^2-2m+4)$
   g) $5(c+6)-4(a^2-2a-1)$
   h) $2\left[3(s-5)-4(s^2+s+3)\right]$
   i) $n^2(n^4-n^2)+4n^3(n+1)$
   j) $2\mathrm{pq}(\mathrm{q}^2-3)-pq(q^2+2)$
   k) $5\mathrm{y}^3+\frac{4y^5}{2y^2}-(7y)y^2$

2. Solve the following equations.
   a) $2y-3=13$
   b) $5a-6=2a-15$
   c) $2k+7=12-3k$
   d) $5(2a-3)=15$
   e) $7x+2=5(x-2)$
   f) $4(2p-3)=3p-27$
   g) $3(x-4)-2(x-5)=6x-2(x-5)$
   h) $\frac{3}{4}m+\frac{5}{6}=5m-\frac{125}{3}$
   i) $5-\frac{3}{g}=35$
   j) $\frac{2y+3}{4}=3y-5$

3. Express the following algebraic fractions as the single friction in its simplest form.
   a) $\frac{s}{6k}-\frac{2-s}{k}$
   b) $\frac{8-p}{3p}-\frac{p+11}{2p}$
   c) $\frac{10-c}{3cd}-\frac{3}{12d}$
   d) $\frac{2}{5h}-\frac{13-h}{15h^2}$

4. Solve the following equations by factoring
   a) $4x^2+x-14=0$
   b) $15x^2-14=29x$
   c) $2x(4x+15)=27$
   d) $8+2x-x^2=0$
   e) $\frac{3x}{x-2}+\frac{1}{x+2}=\frac{4}{x^2-4}$

5. Solve by using the quadratic formula.
   a) $x^2-6x-3=0$
   b) $4y^2+81=36x$
   c) $\frac{5}{3}p^2+3p+1=0$
   d) $\frac{5}{k^2}+\frac{10}{k}+2=0$
   e) $\frac{x+1}{3x+2}=\frac{x-2}{2x-3}$

6. Solve by completing the square.

a) $x^2+6x-7=0$

b) $y^2=8y-11$

c) $3x^2+5x-12=0$

d) $4x^2=12x+11$

7. Convert the following improper fractions to mixed numbers.

a) $\dfrac{x^3+5}{x^3}$

b) $\dfrac{8x^2+2}{x-1}$

c) $\dfrac{x^3-3x^2+4x+1}{x+2}$

e) $\dfrac{4x^2-6x+3}{2x+3}$

8. Find the partial fraction decomposition.

a) $\dfrac{8x-1}{(x-2)(x+3)}$

b) $\dfrac{10-x}{x^2+10x+25}$

c) $\dfrac{2x+3}{(x-1)^2}$

d) $\dfrac{5x^2-4}{x^2(x+2)}$

e) $\dfrac{x^2+10x-36}{x(x-3)^2}$

f) $\dfrac{x^2-x-21}{(x^2+4)(2x-1)}$

g) $\dfrac{3x^3+13x-1}{(x^2+4)^2}$

h) $\dfrac{37-11x}{(x+1)(x^2-5x+6)}$

Answer

1. a) $2x+17y$

b) $\frac{b}{2}(13b+25b)$

c) $3k^3-6k^2+4k-4$

d) $pq\ (5pq-q)$

e) $4y^2-3$

f) $-2(m+10)$

g) $5c+30-4a^2+8a+4$

h) $-2\ (4s^2-s-27)$

i) $n^3\ (n^3+3n+4)$

j) $pq\ (q^2-8)$

k) $2y^3$

2. a) $y=8$

b) $a=-3$

c) $k=1$

d) $a=3$

e) $x=-6$

f) $p=-3$

g) $x=-4$

h) $m=10$

i) $g=-\frac{1}{10}$

j) $y=\frac{23}{10}$

3. a) $\dfrac{7s-12}{6k}$

b) $\dfrac{-17-5p}{6p}$

c) $\dfrac{10-7c}{12cd}$

d) $\dfrac{5h-13}{15h^2}$

4. a) $x_1=\dfrac{7}{4}$ *or* $x_2=-2$

b) $x_1=\dfrac{7}{3}$ *or* $x_2=-\dfrac{2}{5}$

c) $x_1=\dfrac{3}{4}$ *or* $x_2=-\dfrac{9}{2}$

d) $x_1=-2$ *or* $x_2=4$

e) $x_1=0$ *or* $x_2=-\dfrac{1}{3}$

5. a) $x_1=6.46$ *or* $x_2=-0.46$

b) $y_1=\dfrac{9}{2}$ *or* $y_2=\dfrac{9}{2}$

c) $p_1=-0.44$ *or* $p_2=-1.36$

d) $k_1=-0.56$ *or* $k_2=-4.44$

e) $x_1=0.30$ *or* $x_2=-3.30$

6. a) $x_1=1$ *or* $x_2=-7$

b) $y_1=6.24$ *or* $y_2=1.76$

c) $x_1=\dfrac{4}{3}$ *or* $x_2=-3$

d) $x_1=3.74$ *or* $x_2=-0.74$

7. a) $1+\dfrac{5}{x^3}$

b) $8x+\dfrac{8x+2}{x-1}$

c) $x^2-5x+14-\dfrac{27}{x+2}$

d) $2x-6+\dfrac{21}{2x+3}$

8. a) $\dfrac{3}{x-2}+\dfrac{5}{x+3}$

b) $-\dfrac{1}{x+5}+\dfrac{15}{(x+5)^2}$

c) $\dfrac{2}{x-1}+\dfrac{5}{(x-1)^2}$

d) $\dfrac{1}{x}-\dfrac{2}{x^2}+\dfrac{4}{x+2}$

e) $-\dfrac{4}{x}+\dfrac{5}{x-3}+\dfrac{1}{(x-3)^2}$

f) $\dfrac{3x+1}{x^2+4}-\dfrac{85}{17(2x-1)}$

g) $\dfrac{3x}{x^2+4}+\dfrac{x-1}{(x^2+4)^2}$

h) $\dfrac{4}{x+1}+\dfrac{13-4x}{x^2-5x+6}$