CHAPTER 2

Matrices

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Matrices

2.1 Understand matrix

2.1.1 Identify the characteristics and order of a matrix

A matrix is a rectangular array of numbers.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13}\cdots & a_{1n} \\ a_{21} & a_{22} & a_{23}\cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3}\cdots & a_{mn} \end{pmatrix}$$

rows

columns

Numbers in any matrix called as ***elements*** and it will be arranged following by ***row*** and ***column***

The size of the matrix is called its ***order***, and it is denoted by ***rows*** and ***columns***.

Example:

a) $\begin{pmatrix} 6 & 8 \\ 6 & 7 \\ 8 & 3 \end{pmatrix}$ = 3 x 2 matrix

b) $\begin{pmatrix} -2 \\ 6 \\ -5 \end{pmatrix}$ = 3 x 1 matrix

c) $\begin{pmatrix} 2 & 7 & 8 \\ 6 & 1 & 3 \end{pmatrix}$ = 2 x 3 matrix

2.1.2 Write the identity of a matrix

The matrix I is called an identity matrix because IA = A and AI = A for all matrices A. This is similar to the number 1, which is called the multiplicative identity, because 1a = a and a1 = a for all real numbers a.

There is no matrix that works as an identity for matrices of all dimensions. For N×N square matrices there is a matrix IN×N that works as an identity.

Identity matrix (I)

Is also a square matrix where all the main diagonal entries are 1 and all the other entries are zero.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2.1.3 Write the transposition of a matrix

The transpose of a matrix is a new matrix whose rows are the columns of the original (which makes its columns the rows of the original). Here is a matrix and its transpose

If $A = \begin{pmatrix} 1 & 3 & 9 \\ 4 & 2 & 5 \\ 7 & 8 & 3 \end{pmatrix}$ therefore the transpose of matrix A, $A^T = \begin{pmatrix} 1 & 4 & 7 \\ 3 & 2 & 8 \\ 9 & 5 & 3 \end{pmatrix}$

The superscript "T" means "transpose"

2.2 Calculate the operation of matrices

2.2.1 Addition of matrices

If two matrices have the same number of rows and same number of columns, then the matrix sum can be computed

Example:

Let A = $\begin{pmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix}$ and B = $\begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix}$

Addition

A + B = $\begin{pmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix}$ + $\begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix}$

7 + 3 =10

6 + 5 = 11

= $\begin{pmatrix} 10 & 10 & 10 \\ 7 & 9 & 11 \\ 5 & 7 & 5 \end{pmatrix}$

2.2.2 Substraction of matrices

Example:

If A and B have the same number of rows and columns, then A - B is defined as A + (-B)

A – B = $\begin{pmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix}$ - $\begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix}$

2 - 8 =-6

= $\begin{pmatrix} -4 & -6 & -8 \\ 1 & 1 & 1 \\ -3 & -3 & 1 \end{pmatrix}$

2 - 5 = -3

2.2.3 Multiplication of matrices

Example :

Let $A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix}$

Scalar multiplication

To multiply a matrix by a scalar, multiply each element of the matrix by the scalar

Example :

Let $A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix}$

$$yA = y \times \begin{pmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 3y & 2y & y \\ 4y & 5y & 6y \\ y & 2y & 3y \end{pmatrix}$$

Between matrices

If two rectangular matrices are put in order so that the inner dimension is the same in each, then the matrices can be multiplied. The result is (in general) a rectangular matrix:

$$A_{R\times N} * B_{N\times C} = Z_{R\times C}$$

The product AB (if it can be formed) has the same number of rows as A and the same number of columns as B. You can think of this as "canceling" the inner dimension. If the inner dimension cannot be canceled, then the product cannot be formed.

Look at the following product. (For now, ignore how the elements were calculated.)

$$\begin{pmatrix} 6 & 8 \\ 6 & 7 \\ 8 & 3 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 52 & 66 \\ 47 & 60 \\ 31 & 42 \end{pmatrix}$$

This is a 3×2 matrix times a 2×2 matrix. The result is a 3×2 matrix.

Example :

Let $A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix}$

$$A \times B = \begin{pmatrix} \boxed{3 \quad 2 \quad 1} \\ 4 \quad 5 \quad 6 \\ 1 \quad 2 \quad 3 \end{pmatrix} \times \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix}$$

(3x7) + (2x3) + (1x4) = 31

(3x8) + (2x4) + (1x5) = 37

(3x9) + (2x5) + (1x2) = 39

$$= \begin{pmatrix} 31 & 37 & 39 \\ 67 & 82 & 73 \\ 25 & 31 & 25 \end{pmatrix}$$

2.3 Solve simultaneous linear equations up to three variables

2.3.1 Use inverse method

Let say we want to solve this linear equations/system by using Inverse method

$$
\begin{aligned}
7x + \quad 8y + \quad 9z &= \quad p \\
3x + \quad 4y + \quad 5z &= \quad p \\
4x + \quad 5y + \quad 2z &= \quad p
\end{aligned}
$$

Steps:

1. Rewrite in form of Ax = b

2. Find Inverse of matrix $A^{-1} = \frac{1}{|A|} \times Adjoint\ of\ A$

   a. Determinant of matrix A
   b. Minor matrix of A
   c. Cofactor matrix of A
   d. Adjoint matrix of A

3. Solve the variables

   $x = A^{-1}b$

a. Determinant

The determinant of a matrix is a **special number** that can be calculated from the matrix. It tells us things about the matrixes that are useful in system of linear equations, in calculus and more.

The **symbol** for determinant is two vertical lines either side.

$$\boldsymbol{|A|\ means\ the\ determinant\ of\ the\ matrix\ A}$$

Calculating the Determinant

First of all the matrix must be **square** (i.e. have the same number of rows as columns). Then it is just a matter of basic arithmetic. Here is how:

For a 2×2 Matrix

For a 2×2 matrix (2 rows and 2 columns):

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The determinant is:

|A| = ad - bc
"The determinant of A equals a times d minus b times c"

Example:

$$A = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$|\boldsymbol{A}| = \begin{vmatrix} 5 & 6 \\ 7 & 8 \end{vmatrix}$$

$$= 5(8) - 6(7)$$

$$= 40 - 42$$

$$= -2$$

For a 3×3 Matrix

For a 3×3 matrix (3 rows and 3 columns):

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

The determinant is:

|A| = a(ei - fh) - b(di - fg) + c(dh - eg)
"The determinant of A equals ... etc"

It may look complicated, but **there is a pattern**:

$$\mathbf{|A| = a}\begin{vmatrix} \boldsymbol{e} & \boldsymbol{f} \\ \boldsymbol{h} & \boldsymbol{i} \end{vmatrix} \mathbf{- b}\begin{vmatrix} \boldsymbol{d} & \boldsymbol{f} \\ \boldsymbol{g} & \boldsymbol{i} \end{vmatrix} \mathbf{+ c}\begin{vmatrix} \boldsymbol{d} & \boldsymbol{e} \\ \boldsymbol{g} & \boldsymbol{h} \end{vmatrix}$$

Example:

A = $\begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix}$

|A| = 7$\begin{vmatrix} \mathbf{4} & \mathbf{5} \\ \mathbf{5} & \mathbf{2} \end{vmatrix}$**- 8**$\begin{vmatrix} \mathbf{3} & \mathbf{5} \\ \mathbf{4} & \mathbf{2} \end{vmatrix}$**+ 9**$\begin{vmatrix} \mathbf{3} & \mathbf{4} \\ \mathbf{4} & \mathbf{5} \end{vmatrix}$

|A| = 7[8-25]- 8[6-20]+ 9[15-16]

|A| = -119- (-112)+ (-9)

|A| = -16

Or we can use this method

A = $\begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix}$

$$|\boldsymbol{A}| \quad = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix} \begin{matrix} 7 & 8 \\ 3 & 4 \\ 4 & 5 \end{matrix}$$

$$|\boldsymbol{A}| \quad = [(7)(4)(2) + (8)(5)(4) + (9)(3)(5)] - [(9)(4)(4) + (7)(5)(5) + (8)(3)(2)]$$

$$|\boldsymbol{A}| \quad = [56 + 160 + 135] - [144 + 175 + 48]$$

$$|\boldsymbol{A}| \quad = [351] - [367]$$

b. Minor,(M_{ij})

If A is a square matrix, the minor for a_{ij} denoted by $\mathbf{M_{ij}}$ by **eliminating** the i_{th} row and the j_{th} column.

If A = $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ therefore, Minor of A = $\begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$

Example:

If A = $\begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix}$, then the Minor matrix of A

Find determinant of 2x2 matrix

$$M_{11} = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix} = \begin{vmatrix} 4 & 5 \\ 5 & 2 \end{vmatrix} = 4.2 - 5.5 = -17$$

$$M_{12} = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix} = \begin{vmatrix} 3 & 5 \\ 4 & 2 \end{vmatrix} = 3.2 - 5.4 = -14$$

$$M_{13} = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix} = \begin{vmatrix} 3 & 4 \\ 4 & 5 \end{vmatrix} = 3.5 - 4.4 = -1$$

$$M_{21} = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix} = \begin{vmatrix} 8 & 9 \\ 5 & 2 \end{vmatrix} = 8.2 - 9.5 = -29$$

$$M_{22} = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix} = \begin{vmatrix} 7 & 9 \\ 4 & 2 \end{vmatrix} = 7.2 - 9.4 = -22$$

$$M_{23} = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix} = \begin{vmatrix} 7 & 8 \\ 4 & 5 \end{vmatrix} = 7.5 - 8.4 = 3$$

$$M_{31} = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix} = \begin{vmatrix} 8 & 9 \\ 4 & 5 \end{vmatrix} = 8.5 - 9.4 = 4$$

$$M_{32} = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix} = \begin{vmatrix} 7 & 9 \\ 3 & 5 \end{vmatrix} = 7.5 - 9.3 = 8$$

$$M_{33} = \begin{pmatrix} 7 & 8 & 9 \\ 3 & 4 & 5 \\ 4 & 5 & 2 \end{pmatrix} = \begin{vmatrix} 7 & 8 \\ 3 & 4 \end{vmatrix} = 7.4 - 8.3 = 4$$

Therefore the Minor of A = $\begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$

$$M_A = \begin{pmatrix} -17 & -14 & -1 \\ -29 & -22 & 3 \\ 4 & 8 & 4 \end{pmatrix}$$

c. **Cofactor , (C_{ij})**

The cofactor , C_{ij} for the element a_{ij} is defined as $C_{ij} = (-1)^{i+j} M_{ij}$

Example :

$M_A = \begin{pmatrix} -17 & -14 & -1 \\ -29 & -22 & 3 \\ 4 & 8 & 4 \end{pmatrix}$ therefore the Cofactor of A = $\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}$

$$C_{12} = (-1)^{1+2} M_{12}$$

$$C_{12} = (-1)^{1+2} (-14)$$

$$C_{12} = (-1)^{3} (-14)$$

$$C_{12} = (-1)\ (-14)$$

$$C_{12} = 14$$

Or we can simply apply the “Checkerboard” on the Minor matrix.

$$\begin{pmatrix} -17 & -14 & -1 \\ -29 & -22 & 3 \\ 4 & 8 & 4 \end{pmatrix} \Rightarrow \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} \Rightarrow \begin{pmatrix} -17 & 14 & -1 \\ 29 & -22 & -3 \\ 4 & -8 & 4 \end{pmatrix}$$

Minor Matrix | Checkerboard | Cofactor Matrix

Therefore the Cofactor of A = $\begin{pmatrix} -17 & 14 & -1 \\ 29 & -22 & -3 \\ 4 & -8 & 4 \end{pmatrix}$

<u>Adjoint, A_{ij}</u>

The Adjoint , A_{ij} for the element a_{ij} is defined as $A_{ij} = (C_{ij})^T$

Example

Cofactor of A = $\begin{pmatrix} -17 & 14 & -1 \\ 29 & -22 & -3 \\ 4 & -8 & 4 \end{pmatrix}$ therefore the Adjoint matrix of A = $\begin{pmatrix} -17 & 29 & 4 \\ 14 & -22 & -8 \\ -1 & -3 & 4 \end{pmatrix}$

Cramer's rule

Cramer's Rule for 3 Equations
Given a triple of simultaneous equations
a1x + b1y + c1z = d1
a2x + b2y + c2z = d2
a3x + b3y + c3z = d2
then x, y, and z can be found by

Let A = $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$

Steps:

1. Find the Determinant of A
2. Setup the matrix of A_1 and find the determinant of A_1

   $A_1 = \begin{pmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{pmatrix}$
3. Setup the matrix of A_2 and find the determinant of A_2

   $A_2 = \begin{pmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{pmatrix}$
4. Setup the matrix of A_3 and find the determinant of A_3

   $A_3 = \begin{pmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{pmatrix}$
5. Therefore

$$x = \frac{|A_1|}{|A|} \qquad y = \frac{|A_2|}{|A|} \qquad z = \frac{|A_3|}{|A|}$$

Example

$$\begin{array}{rcrcrcr} -2x_1 & - & 3x_2 & - & x_3 & = & 1 \\ x_1 & + & 2x_2 & - & x_3 & = & 4 \\ -2x_1 & - & x_2 & + & x_3 & = & -3 \end{array}$$

Write in the form of , Ax = B

$$\begin{bmatrix} -2 & 3 & -1 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$$

A X B

Find the determinant of A

$$A = \begin{bmatrix} -2 & 3 & -1 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix}$$

$$|A| = -2\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$$

$$= -2(1) \quad -3(-1) \quad -1(3)$$

$$|A| = -2$$

Find the determinant of A_1, A_2 and A_3

$$A_1 = \begin{bmatrix} 1 & 3 & -1 \\ 4 & 2 & -1 \\ -3 & -1 & 1 \end{bmatrix}, \quad |A_1| = 1\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} - 3\begin{vmatrix} 4 & -1 \\ -3 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 4 & 2 \\ -3 & -1 \end{vmatrix}$$

$$= 1(1) \ -3(1) \ -1(2)$$

$$= -4$$

$$A_2 = \begin{bmatrix} -2 & 1 & -1 \\ 1 & 4 & -1 \\ -2 & -3 & 1 \end{bmatrix}, \quad |A_2| = -2\begin{vmatrix} 4 & -1 \\ -3 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 4 \\ -2 & -3 \end{vmatrix}$$

$$= -2(1) - 1(-1) - 1(5)$$

$$= -6$$

$$A_2 = \begin{bmatrix} -2 & 3 & 1 \\ 1 & 2 & 4 \\ -2 & -1 & -3 \end{bmatrix}, \quad |A_3| = -2\begin{vmatrix} 2 & 4 \\ -1 & -3 \end{vmatrix} - 3\begin{vmatrix} 1 & 4 \\ -2 & -3 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$$

$$= -2(-2) - 3(5) + 1(3)$$

$$= -8$$

Therefore, from the Cramer's rule

$$x_1 = \frac{|A_1|}{A} = \frac{-4}{-2} = 2$$

$$x_2 = \frac{|A_2|}{A} = \frac{-6}{-2} = 3$$

$$x_3 = \frac{|A_3|}{A} = \frac{-8}{-2} = 4$$

$$\therefore \ \textit{nilai bagi} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

Exercises

1. Solve the following equation. Given:

$$A=\begin{pmatrix}-1 & 2\\ 3 & 0\end{pmatrix} \qquad B=\begin{pmatrix}2 & -3\\ 1 & 4\end{pmatrix} \qquad C=\begin{pmatrix}0 & -2\\ -3 & -1\end{pmatrix}$$

   i) $A+(B-C)$
   ii) $(A+B)-C$

2. Given $A = \begin{pmatrix}-4 & 6\\ 5 & 3\end{pmatrix}$, $B = \begin{pmatrix}1 & 3\\ 2 & 1\end{pmatrix}$ and $C = \begin{pmatrix}4 & 2\\ 1 & 6\end{pmatrix}$, find the values of

   i) (A + B) – C

   ii) (C + B) A

   iii) AC – BA

3. Given $A = \begin{pmatrix}3\\ 2\\ 4\end{pmatrix}$ $B = \begin{pmatrix}2 & 1 & 3\\ 4 & 1 & 2\\ 3 & 2 & 4\end{pmatrix}$ $C = \begin{pmatrix}3 & 5 & 3\\ 2 & 1 & 1\\ 4 & 3 & 1\end{pmatrix}$, find

   a) *CA + A*
   b) *BC – B*
   c) *2B – C*

4. If $A=\begin{pmatrix}2 & 9 & -4\\ 6 & 1 & -2\end{pmatrix}$ $B=\begin{pmatrix}4 & 0 & -4\\ -3 & 4 & 5\end{pmatrix}$ $C=\begin{pmatrix}4 & -2\\ 7 & 1\\ 3 & 6\end{pmatrix}$

   i) $3A-B$
   ii) $4B^T+C$
   iii) BC

4. Given the minor matrix of $M = \begin{pmatrix} -1 & -3 & 5 \\ 4 & 7 & 2 \\ -2 & 6 & 1 \end{pmatrix}$ and its determinant is given as $|M| = -8$, evaluate

i) Adjoint of matrix *M, Adj (M)*

ii) Inverse of matrix *M, M^{-1}*

5. Solve the following equations by using Inverse Method

a)
$$\begin{aligned} x + 2y - 4z &= 5 \\ -x - y + 6z &= -6 \\ 2x + 3y - 12z &= 9 \end{aligned}$$

b)
$$\begin{aligned} x + 2y - 4z &= -4 \\ 2x + 5y - 9z &= -10 \\ 3x - 2y + 3z &= 11 \end{aligned}$$

c)
$$\begin{aligned} 2x + 4y + 7z &= 26 \\ 5x - 6y + 9z &= -169 \\ 3x + 6y - 5z &= 225 \end{aligned}$$

d)
$$\begin{aligned} x + 5y + 4z &= 99 \\ 6x - y + 7z &= 75 \\ 4x + 6y + z &= 125 \end{aligned}$$

6. Solve the following equations by using Cramer's Rule

a)
$$\begin{aligned} -x+2y+4z&=-8\\ 3x+2y-z&=-2\\ x+y+2z&=2 \end{aligned}$$

b)
$$\begin{aligned} x+y+2z&=4\\ 2x+3y+6z&=10\\ 3x+6y+10z&=14 \end{aligned}$$

c)
$$\begin{aligned} x+2y-3z&=-1\\ -2x+3y+2z&=2\\ 3x-4y+z&=-6 \end{aligned}$$

d)
$$\begin{aligned} 3x-4y+5z&=18\\ -9x+8y+7z&=-13\\ 5x-7y+10z&=33 \end{aligned}$$