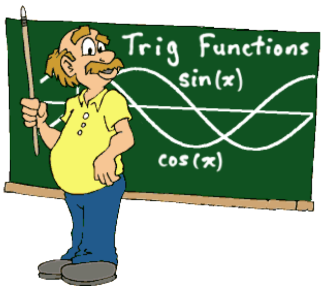
CHAPTER 2 : TRIGONOMETRY

Trigonometry (from Greek trigōnon, "triangle" + metron, "measure"[1]) is a branch of mathematics that studies relationships involving lengths and angles of triangles. The field emerged during the 3rd century BC from applications of geometry to astronomical studies.[2]



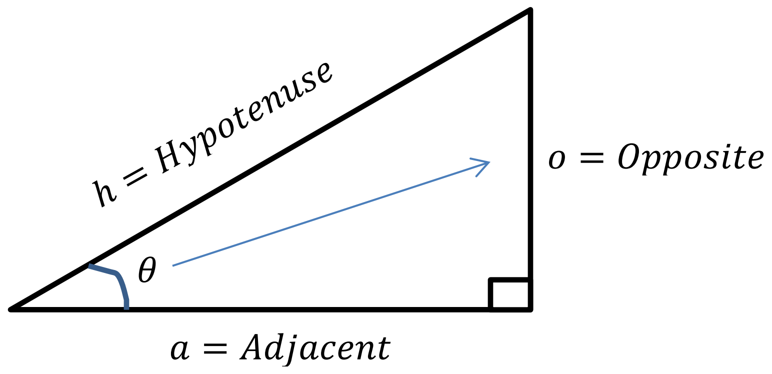
The 3rd-century astronomers first noted that the lengths of the sides of a right-angle triangle and the angles between those sides have fixed relationships: that is, if at least the length of one side and the value of one angle is known, then all other angles and lengths can be determined algorithmically. These calculations soon came to be defined as the trigonometric functions and today are pervasive in both pure and applied mathematics: fundamental methods of analysis such as the Fourier transform, for example, or the wave equation, use trigonometric functions to understand cyclical phenomena across many applications in fields as diverse as physics, mechanical and electrical engineering, music and acoustics, astronomy, ecology, and biology. Trigonometry is also the foundation of the practical art of surveying.

Trigonometry is most simply associated with planar right-angle triangles (each of which is a two-dimensional triangle with one angle equal to 90 degrees). The applicability to non-right-angle triangles exists, but, since any non-right-angle triangle (on a flat plane) can be bisected to create two right-angle triangles, most problems can be reduced to calculations on right-angle triangles. Thus the majority of applications relate to right-angle triangles. One exception to this is spherical trigonometry, the study of triangles on spheres, surfaces of constant positive curvature, in elliptic geometry (a fundamental part of astronomy and navigation). Trigonometry on surfaces of negative curvature is part of hyperbolic geometry.

Trigonometry basics are often taught in schools, either as a separate course or as a part of a precalculus course.

2.1 Fundamental of trigonometric functions

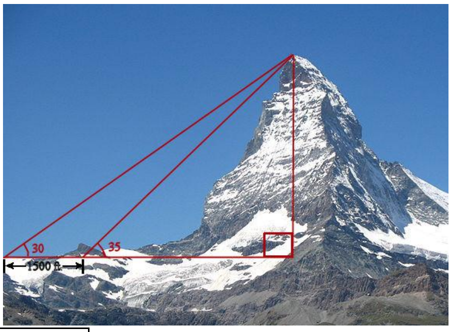
Right Angled Triangle



2.1.1 Sine, cosine, tangent, secant, cosecant and cotangent

Sine, Cosine and Tangent

$sin\ \theta = \frac{o}{h}$	
$cos\ \theta = \frac{a}{h}$	
$tan\ \theta = \frac{o}{a}$	$tan\ \theta = \frac{sin\ \theta}{cos\ \theta}$



Cosecant, Secant and Cotangent

$csc\ \theta = \frac{1}{sin\ \theta}$	$csc\ \theta = \frac{h}{o}$	
$sec\ \theta = \frac{1}{cos\ \theta}$	$sec\ \theta = \frac{h}{a}$	
$cot\ \theta = \frac{1}{tan\ \theta}$	$cot\ \theta = \frac{a}{o}$	$cot\ \theta = \frac{cos\ \theta}{sin\ \theta}$

Example:

1. Given $\sin x = \frac{7}{25}$ and $\cos x = -\frac{24}{25}$, determine the value of each of the following.

a) $\tan x$

$$= \frac{\sin x}{\cos x}$$
$$= \frac{7}{25} \div \left(-\frac{24}{25}\right)$$
$$= -\frac{7}{24}$$

b) $\cos x$

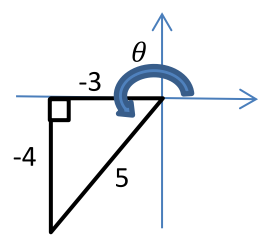
c) $\sec x$

d) $cosec\ x$

2. Given $\sin\theta = -\frac{4}{5}$ and $90° < \theta < 270°$, find the value of each of the following without using calculator.

$\tan\theta$

$$= \frac{-4}{-3}$$
$$= \frac{4}{3}$$



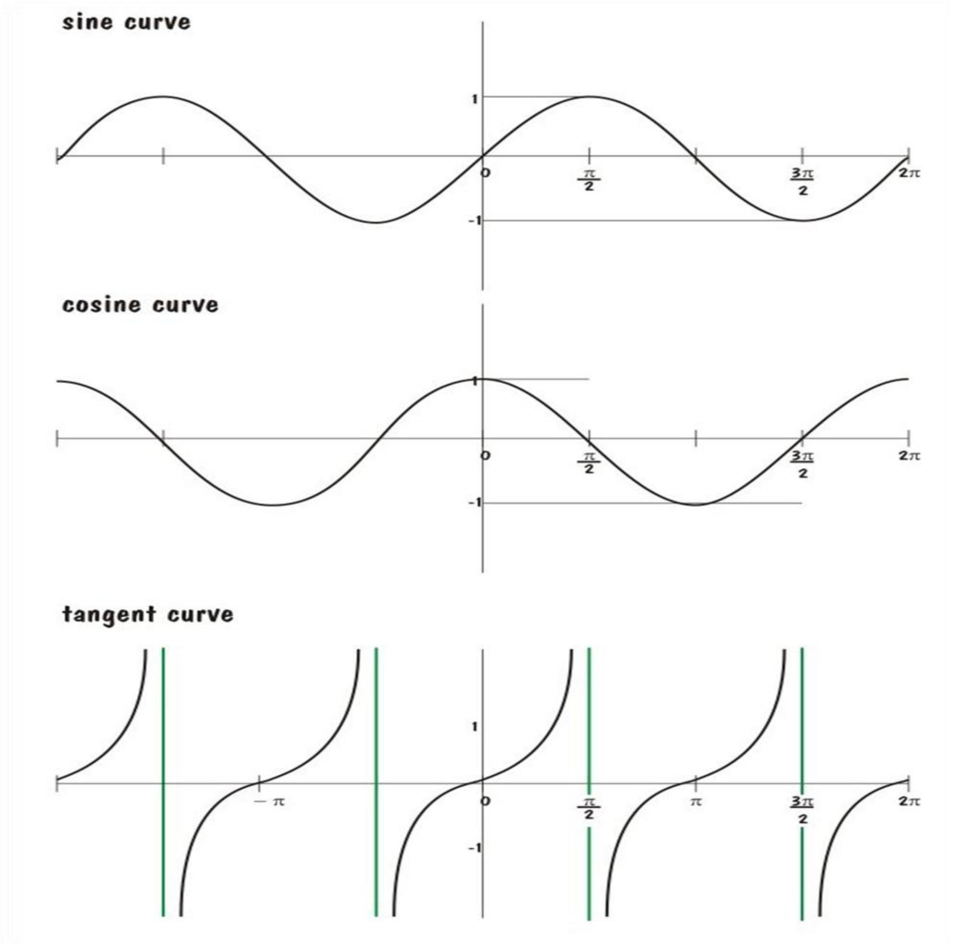
a) $\cos\theta$

b) $\cot\theta$

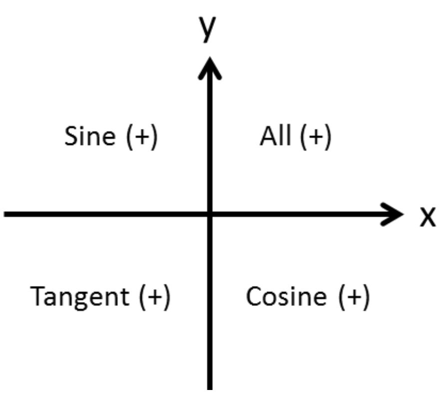
c) $\sec\theta$

d) $\text{cosec}\,\theta$

2.1.2 Graphs of Sine, Cosine and Tangent



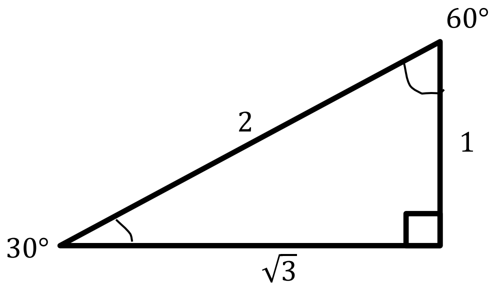
2.1.3 Signs of trigonometric functions

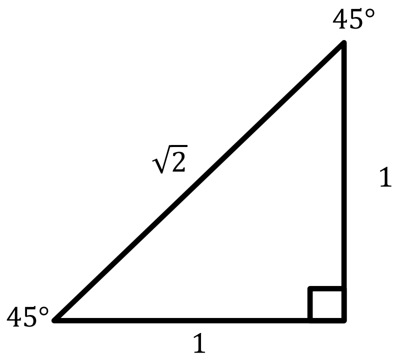


2.1.4 Six Trigonometric functions of Any Angles

Trigonometric ratios of special angles:

$0°, 30°, 45°, 60°$ and $90°$

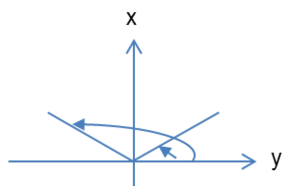
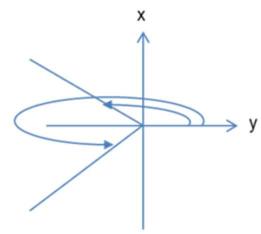




θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

Example:

3. Solve the following trigonometric equations for $0° < x < 360°$.

$\sin x = 0.4$ $x = sin^{-1}0.4$ Basic angle $= 23°35'$ $= 23°35', 180° - 23°35'$ $= 23°35', 156°25'$  	$\cos 2x = -0.8090$ $2x = -cos^{-1}0.8090$ $= -36°$ Basic angle $= 36°$ $2x = 180° - 36°, 180° + 36°,$ $540° - 36°, 540° + 36°$ $= 144°, 216°, 504°, 576°$ $x = 72°, 108°, 252°, 288°$ $0° \leq x \leq 360°$ $0° \leq 2x \leq 720°$ 

a) $\cos\ x = 0.7431$

b) $\sin 2x = 0.5050$

c) $\cot\ x = 1.6$

d) $\tan 2x = -0.7813$

e) $\sec\ x = -1.095$

f) $cos\frac{1}{2}x = 0.6637$

Example:

4. Find the value of x which satisfy the following equations for $0° \leq x \leq 360°$.

$4sin^2 x + 3\sin x = 0$ $sin\ x\ (4\sin x + 3) = 0$ $\sin x = 0$ $x = 0°, 180°, 360°$ $4\sin x + 3 = 0$ $\sin x = -\frac{3}{4}$ $x = 228°35', 311°25'$ $\therefore x = 0°, 180°, 228°35', 311°25', 360°$	$\sin x = 3\tan x$ $\sin x = 3\left(\frac{\sin x}{\cos x}\right)$ $\sin x\cos x - 3\sin x = 0$ $\sin x\ (\cos x - 3) = 0$ $\sin x = 0$ $x = 0°, 180°, 360°$ $\cos x - 3 = 0$ $\cos x = 3$ (No solution because $-1 \leq \cos x \leq 1$) $\therefore x = 0°, 180°, 360°$

a) $\sin x - 2\sin x\cos x = 0$
b) $4\ cosx = \sec x$
c) $2\sin x + 5\cos x = 0$
d) $2\sin x\cos x = 3\ cos^2\ x$
e) $4\sin x - 3\ cosec\ x = 0$

2.2 Understand trigonometric equations and identities.

2.2.1 Trigonometric basic identities

$sin^2\theta + cos^2\theta = 1$
$1 + tan^2\theta = sec^2\theta$
$cos^2\theta + 1 = cosec^2\theta$

Example:

5. Solve the following trigonometric equations for $0° \leq x \leq 360°$.

$$2\, sin^2 + \cos x - 1 = 0$$
$$2\,(1 - cos^2 x) + \cos x - 1 = 0$$
$$2cos^2 x - \cos x - 1 = 0$$
$$(2\cos x + 1)(\cos x - 1) = 0$$

$$2\cos x + 1 = 0$$
$$\cos x = -\frac{1}{2}$$
$$x = 120°, 240°$$

$$\cos x - 1 = 0$$
$$\cos x = 1$$
$$x = 0°, 360°$$

Thus, $x = 0°, 120°, 240°, 360°$

a) $3\, sin^2 x - \cos x - 1 = 0$

b) $5cos^2 x + 2\sin x - 2 = 0$

c) $sec^2 x - \tan x - 7 = 0$

d) $tan^2\, x - 3\sec x - 9 = 0$

2.2.2 State the Compound Angle formulae

$\sin(A \pm B) = sin\ A\ cos\ B \pm sin\ Bcos\ A$
$cos\ (A \pm B) = cos\ A\ cos\ B \mp sin\ A\ sin\ B$
$tan\ (A \pm B) = \dfrac{tan\ A \pm \tan B}{1 \mp \tan A \tan B}$

2.2.3 State the Double Angle Formulae

$\sin 2A = 2\sin A\ \cos A$
$\cos 2A = cos^2A + sin^2A$
$\tan 2A = \dfrac{2\tan A}{1 - tan^2A}$

Example:

6. Without using calculator, determine the values of the following.

$\sin 36° \cos 54° + \cos 36° \sin 54°$
$= \sin(36° + 54°)$
$= \sin 90°$
$= 1$

a) $\sin 50° \cos 40° + \cos 50° \sin 40°$

b) $\cos 72° \cos 12° + \sin 72° \sin 12°$

c) $\dfrac{\tan 63° - \tan 18°}{1 + \tan 63° \tan 18°}$

7. Without using a calculator, find the values of $\sin 2\theta, \cos 2\theta$ and $\tan 2\theta$.

a) Given $\sin\theta = \frac{2}{3}$ and $90° < \theta < 180°$.

b) Given $\tan\theta = -\frac{3}{5}$ and $180° < \theta < 360°$.

Example:

8. Find the values of x between $0°$ and $360°$ inclusive, which satisfy each equation.

$$\cos 2x - \cos x - 2 = 0$$

$$\cos 2x - \cos x - 2 = 0$$
$$2\,cos^2 x - 1 \cos x - 2 = 0$$
$$2\,cos^2 x - \cos x - 3 = 0$$
$$(2\cos x - 3)\,(\cos x + 1) = 0$$

$2\cos x - 3 = 0$ or $\cos x + 1 = 0$

$\cos x = \frac{3}{2}$; $\cos x = -1$, $x = 180°$

(No solution because $-1 \leq \cos x \leq 1$)

Thus , $x = 180°$

a) $2\cos 2x + \sin x + 1 = 0$

b) $2\cos 2x - cos^2 x + 1 = 0$

c) $2\sin 2x = 3\sin x$