CHAPTER 1 : COMPLEX NUMBER

Formula:

$i^1 = i$

$i^2 = -1$

Tip:

$(-1)^{\text{odd number}} = -1$

$(-1)^{\text{prime number}} = 1$

Example:

1. i^{112}

Step by step

Step 1

112/2 =56

Step 2

a) $i^{112} = (i^2)^{56}$
$= (-1)^{56}$
$= 1$

Prime number

1. $i^{90} = (i^2)^{45}$
$= (-1)^{45}$
$= -1$

Odd number

2. $i^{8} = (i^2)^{4}$

3. $i^{20} = (i^2)^{10}$

4. $i^{114} =$

5. $i^{6} =$

6. $i^{34} =$

TIP : if power of i is odd

a) Subtract power of i with 1.
b) After that divide with 2

Example :

1. i^{103}

Step by step

Step 1

103 – 1 = 102

Step 2

102/2 =51

Step 3

$i^{103} = (i^2)^{51} \cdot (i)^1$
$= (-1)^{51} \cdot i$
$= (-1)i$
$= -i$

2. $i^{91} = (i^2)^{45} \cdot (i)^1$
$= (-1)^{45} \cdot i$
$= -1i$
$= -i$

3. $i^{9} = (i^2)^{4} \cdot (i)^1$

4. $i^{21} =$

5. $i^{115} =$

6. $i^{7} =$

7. $i^{35} =$

Formula:

$$i = \sqrt{-1}$$
$$i^2 = -1$$

Example:-

1. $\sqrt{-18} = (\sqrt{9}).(\sqrt{2}).(\sqrt{-1})$

$= 3\,i\,\sqrt{2}$

2. $\sqrt{-9} = \sqrt{9(-1)}$

$= \sqrt{9i^2}$

$= 3i$

Exercise :

Simplify

a) i^{20}	b) i^{75}	c) i^{34}	d) i^{104}	e) i^{203}	f) i^{65}
g) $\sqrt{-20}$	h) $\sqrt{-32}$	i) $\sqrt{-144}$	j) $\sqrt{-36}$	k) $\sqrt{-24}$	l) $\sqrt{-81}$

Answer :

a) 1	b) -i	c) -1	d) 1	e) -i	f) i
g) $2\sqrt{5}\ i$	h) $4\sqrt{2}\ i$	i) 12i	j) 6i	k) $2\sqrt{6}\ i$	l) 9i

ADDITION AND SUBTRACTION

Example:-

1. $5i^7 - 2i^6 \quad = 5i(i^2)^3 - 2(i^2)^3$

   $= 5i(-1)^3 - 2(-1)^3$

   $= -5i + 2$

2. $\sqrt{-81} + \sqrt{-36} = \sqrt{-1}.\sqrt{81} + \sqrt{-1}.\sqrt{36}$

   $= 9i + 6i$

   $= 15i$

3. $\sqrt{-25} - 3i^5 \quad = \sqrt{-1}.\sqrt{25} - 3i(i^2)^2$

   $= 5i - 3i(-1)^2$

   $= 5i - 3i$

   $= 2i$

Exercise :

1. $2i^4 + 5i^{12}$
2. $-3i^{15} + 4i$
3. $6i^7 - 3i^3$
4. $-5i^9 - 6i^4$
5. $\sqrt{-169} + \sqrt{-121}$
6. $\sqrt{-49} - \sqrt{-25}$
7. $\sqrt{-4} + 2i^3$
8. $-4i^{23} - \sqrt{-25}$
9. $6i^5 - \sqrt{-36}$
10. $\sqrt{-144} - 3i^7$

MULTIPLICATION AND DEVIATION

Example:-

1. $2i^3 \times 4i^8 = (2\times 4)(i^{3+8})$ (Using index law)

   $=8i^{11}$

   $=8i(i^2)^5$

   $=8i(-1)^5$

2. $\sqrt{-9}\times\sqrt{-36} = \sqrt{-1}.\sqrt{9}\times\sqrt{-1}.\sqrt{36}$

   $=3i\times 6i$ (Using index law)

   $= 18i^2$

   $=18(-1)$

   $=-18$

3. $\sqrt{-25}\times 2i^3 = \sqrt{-1}.\sqrt{25}\times 2i(i^2)$

   $=5i\times 2i(-1)$

   $=5i\times(-2i)$

   $=-10i^2$

   $=-10(-1)$

   $=10$

4. $12i^6 \div 6i^3 = \frac{12}{6}(i^{6-3})$

   $=2i^3$

   $=2i(i^2)$

   $=2i(-1)$

   $=-2i$

Exercise :-

1. $5i^7(\sqrt{-36}+3)$

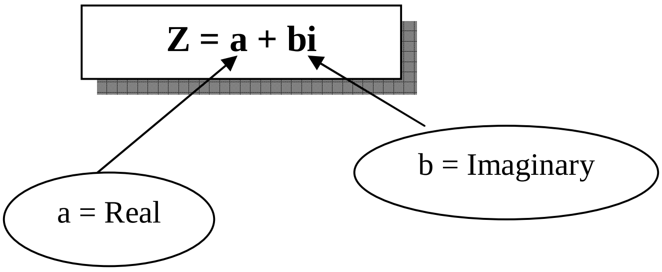
2. $\sqrt{-36}i\times 4i^8(2i^9+3i^4)$

3. $i^3(\sqrt{-49}+4i^2)$

4. $(\sqrt{-144}.\sqrt{-36}.2i^4.(-4i^7)$

Complex Plane :

Formula:



ADDITION AND SUBTRACTION

Example :

a) $(5 + 3i) + (7 + 5i)$

$= 5 + 7 + 3i + 5i$

$= 12 + 8i$

b) $(6 - 2i) + (5 + 4i)$

c) $(5 - 2i) - (4 - 5i)$

d) $(4 + 3i) - (2 - i)$

Exercise :

a) $(2 + 3i) + (3 - 2i)$ b) $(4 - 3i) - (2 + 2i)$ c) $(5 + 2i) + (2 - 3i)$

d) $(10 + 2i) - (4 - 3i)$ e) $(13 - 2i) - (5i)$ d) $(14 - i) + (3i)$

if z = (4 + 3i) , w = (2 - 4i) find

i. $\bar{z} + 2w$

ii. $4w - 5z$

answer :

a) 5 + i b) 2 – 5i c) 7 – i d) 6 + 5i e)13 – 7i

MULTIPLICATION

Tips : Change i^2 to (-1)

Example :

a) $(3 + 3i)(2 + 5i)$

$= 6 + 15i + 6i + 15i^2$

$= 6 + 15i + 6i + 15(-1)$

$= 6 - 15 + 15i + 6i$

$= -9 + 21i$

b) $(2 + 3i)(2 - 7i)$

$= 4 - 14i + 6i - 21i^2$

$= 4 - 14i + 6i - 21(-1)$

$= 4 + 21 - 14i + 6i$

$= 25 - 8i$

Exercise :

a) $(4 + 2i)(3 + 5i)$ *b)* $(5 - 2i)(3 + 4i)$ *c)* $(9 - 2i)(3 - 4i)$ *d)* $(5 + 4i)(6 - 2i)$

e) if z = (4 + 5i) , w = (6 - 4i) find

i. $\bar{z} \times 2w$

ii. $2z \times w$

Answer :

a) $2 + 26i$ *b)* $23 + 14i$ *c)* $19 - 42i$ *d)* $38 + 14i$

DIVISION

Conjugate ($\bar{Z}$) for :

Z = a + bi is $\bar{Z}$ = a – bi

Z = a – bi is $\bar{Z}$ = a + bi

Example :

a) $\frac{5+2i}{2-3i} = \frac{5+2}{2-3i} \times \frac{2+3i}{2+3i}$

$= \frac{(5+2i)(2+3i)}{(2-3i)(2+3i)}$

$= \frac{10+15i+4i+6i^2}{4+6i-6i-9i^2}$

$= \frac{10+15i+4i+6(-1)}{4+6i-6i-9(-1)}$

$= \frac{10-6+15i+4i}{4+9+6i-6i}$

$= \frac{4+19i}{13}$

$= \frac{4}{13} + \frac{19}{13}i$

b) $\frac{2-3i}{4-5i}$

$= \frac{2-3i}{4-5i} \times \frac{4+5i}{4+5i}$

$= \frac{(2-3i)(4+5i)}{(4-5i)(4+5i)}$

$= \frac{8+10i-12i-15i^2}{16+20i-20i-25i^2}$

$= \frac{8-2i-15(-1)}{16-25(-1)}$

$= \frac{8+15-2i}{16+25}$

$= \frac{23-2i}{41}$

$= \frac{23}{41} - \frac{2i}{41}$

Exercise :

a) (4 + 2i) ÷ (2 + 5i) b) (5 – 2i) ÷ (3 - 4i)

c) if z = (4 + 3i) , w = (2 - 4i) find z ÷ 2w

Answer :

a) $\frac{18}{29} + \frac{16i}{29}$ b) $\frac{23}{25} + \frac{14i}{25}$ c) $\frac{-1}{10} + \frac{11i}{20}$

EQUALITY

If
a + bi = c + di

a = c and b = d

Example :

Find value of m and n

i) m + ni = (3 + 3i) (2 + 5i)

= 6 + 15i + 6i + 15i²

= 6 + 15i + 6i + 15(-1)

= 6 – 15 +15i + 6i

= -9 + 21i

thus m = -9 and n = 21

ii)
Calculate the equation $x+2+yi=-y-x^2i$ for x and y value.

$x+2+yi=-y-x^2i$

$x+2=-y$ ---------------------- equation 1 (real number)

$y=-x^2$ ---------------------- equation 2 (imaginary)

Substitut equation 2 into equation1

$x+2=-(-x^2)$

$x+2=x^2$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x_1=2$ and $x_2=-1$

$y_1 = -(2^2) = -4$ $y_2 = -(-1)^2 = -1$

iii) $(x+yi)(-9+2i) = (4-7i)$

$$(x+yi) = \frac{(4-7i)}{(-9+2i)}$$

$$(x+yi) = \frac{(4-7i)}{(-9+2i)} \times \frac{-9-2i}{-9-2i}$$

$$(x+yi) = \frac{(4-7i)}{(-9+2i)} \frac{(-9-2i)}{(-9-2i)}$$

$$(x+yi) = \frac{-36-8i+63i+14i^2}{81+18i-18i-4i^2}$$

$$(x+yi) = \frac{-36+55i+14(-1)}{81-4(-1)}$$

$$(x+yi) = \frac{-50+55i}{85}$$

$$(x+yi) = \frac{-10}{17} + \frac{11i}{17}$$

Thus $x = \frac{-10}{17}$ and $y = \frac{11}{17}$

Exercise:

Find value of a and b

a) (4+ 2i) (a + 2bi) = 2 – 3i

b) (2a -3bi) + (5 + 2i) = 7 + 7i

c) (2a + 2bi) = 10

d) (10a -2bi)- (5a + 2bi)= 4i

Answer:

a) a = 1/10 b = -2/5 b) a = 1 b = -5/3 c) a = 5 b = 0

d) a = 0 b = -1

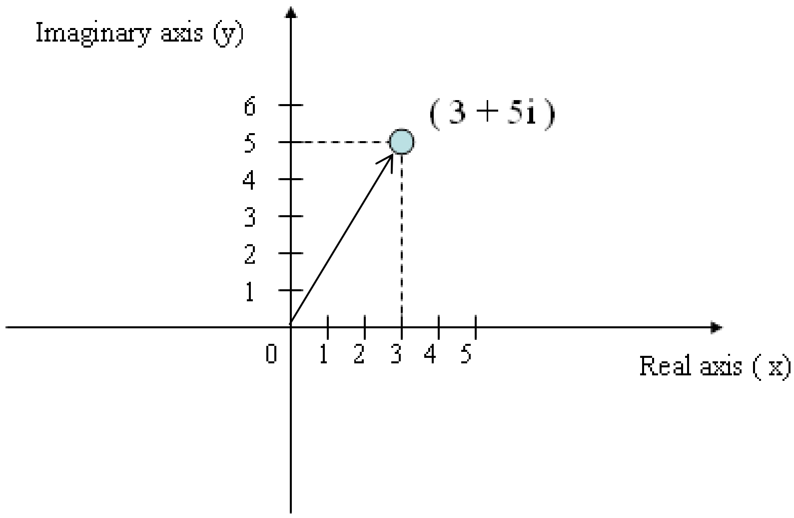
ARGAND DIAGRAMS

Z = x + yi

Example :

Plot this complex number equation at argand diagrams :

a) Z = 3 + 5i



Exercise:

a) 4 + 5i b) -3 + 2i c) -4 – 6i d) 6 – 4i e) 7 f) 4i

ADDITION AND SUBTRACTION USING ARGAND DIAGRAM

Example:-

Given $Z_1 = 4 + 3i$ and $Z_2 = 2 - 4i$, find the following using argand diagram:-

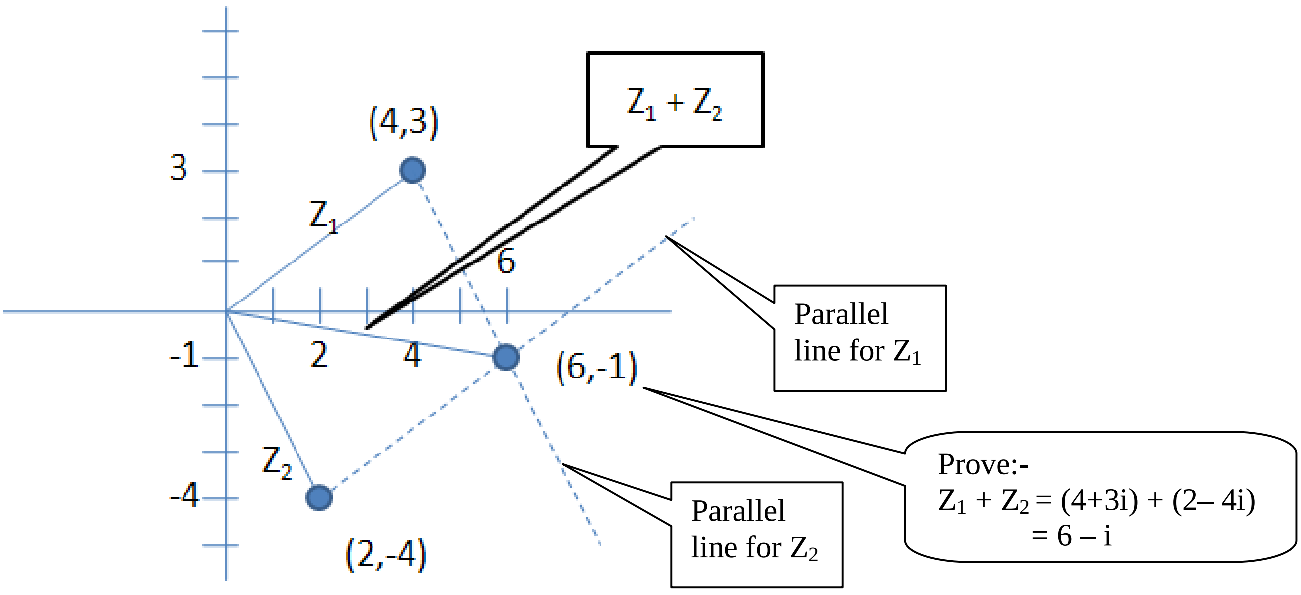
i) $Z_1 + Z_2$
ii) $Z_1 - Z_2$

Solution:

i) $Z_1 + Z_2$

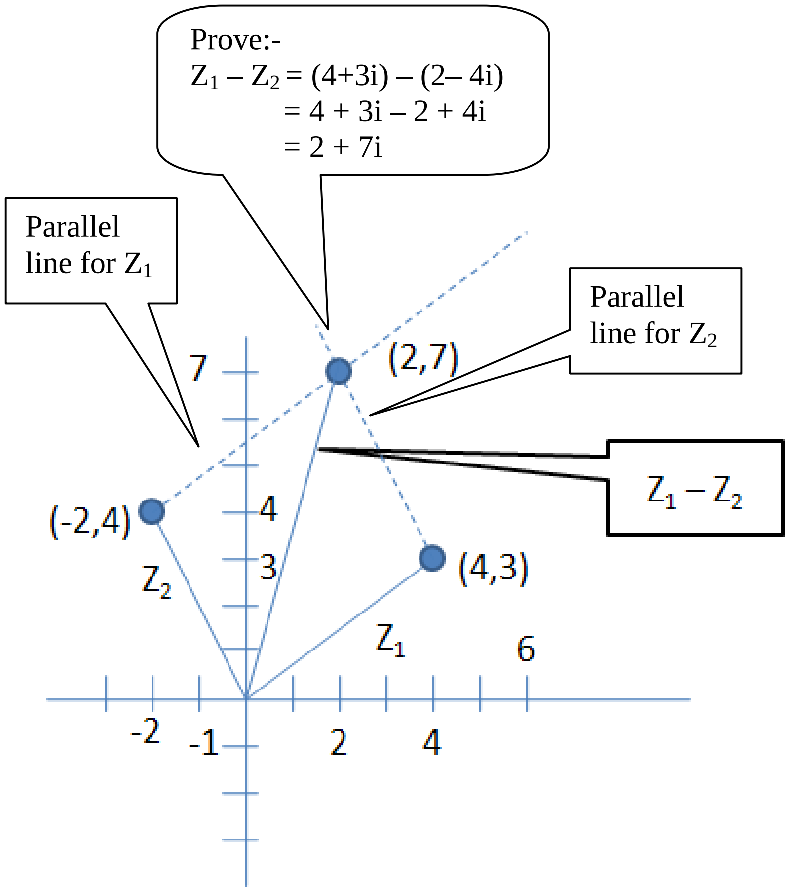
Step by step:-

1. Plot Z_1 and Z_2 using graph paper.
2. Draw parallel line for Z_1 and Z_2.
3. Find the intersection point.



ii) $Z_1 - Z_2 = (4 + 3i) - (2 - 4i)$
$= 4 + 3i - 2 + 4i$

In this case $Z_1 = 4 + 3i$ but $Z_2 = -2 + 4i$



Exercise :-

Solve the following using argand diagram :-

Given $Z_1 = -2 + 3i$ $Z_2 = -5 - 2i$ $Z_3 = 3 + 4i$

a) $Z_1 + Z_3$ b) $Z_3 - Z_2$ c) $Z_2 + Z_1$ d) $Z_2 - Z_1$

MODULUS AND ARGUMENT

Modulus =|z|=r

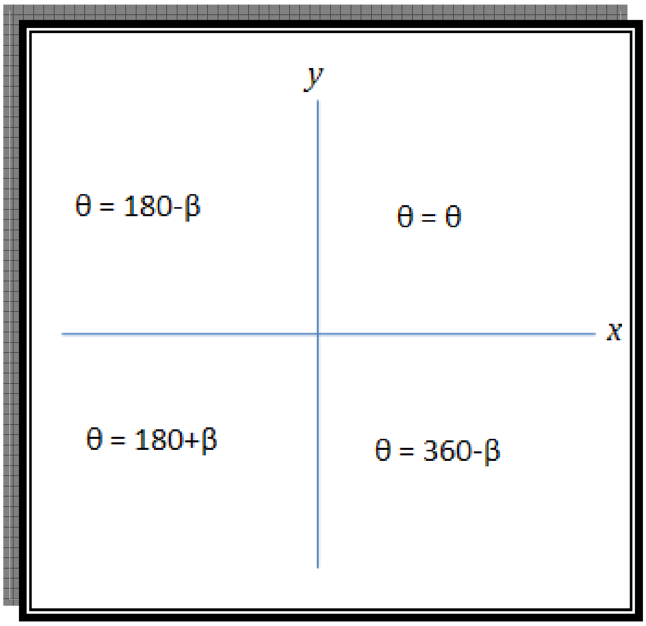
r = $\sqrt{a^2+b^2}$ $\sqrt{(\mathbf{x}^2+\mathbf{y}^2)}$

Argument = θ

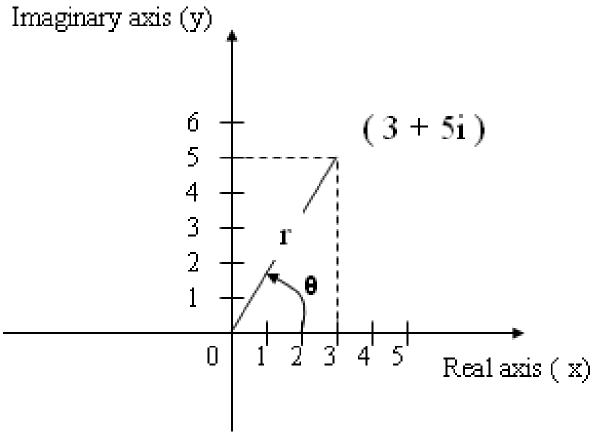
$$\theta = \tan^{-1}\frac{y}{x}$$

Tips : Angle θ

- From (+) real axis to r
- Anticlockwise

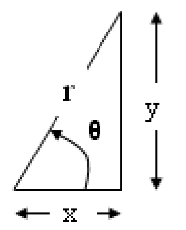


FIND ARGUMENT



1st quadrant

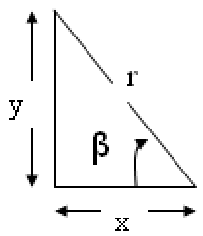
$$\theta = \tan^{-1} \frac{y}{x}$$

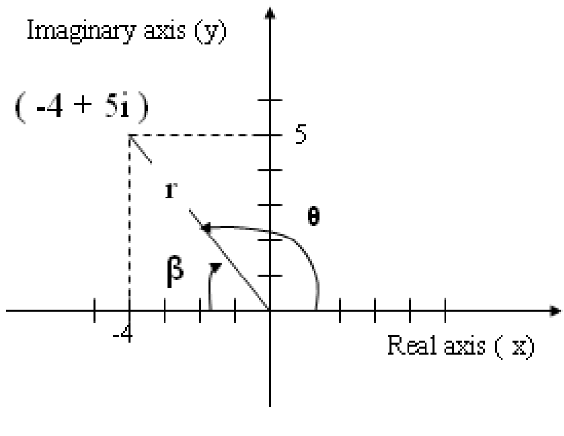


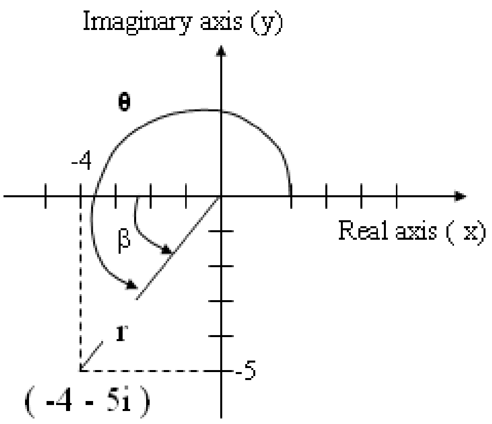
2nd quadrant

$$\beta = \tan^{-1} \frac{y}{x}$$

$$\theta = 180^{o} - \beta$$



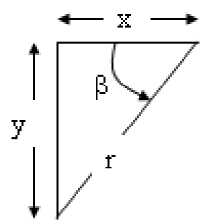




3nd quadrant

$$\beta = \tan^{-1} \frac{y}{x}$$

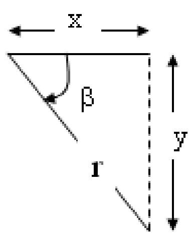
$$\theta = 180^{o} + \beta$$

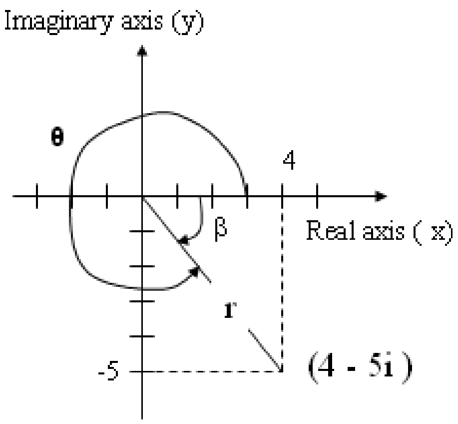


4nd quadrant

$$\beta = \tan^{-1} \frac{y}{x}$$

$$\theta = 360^{o} - \beta$$





Note : If x = 0 , θ = 90°	If y = 0 , θ = 0°
Example : if z = 2i Thus θ = 90°	Example : if z = 3 Thus θ = 0°

Example :

Fine argument and modulus

a) $z = 12 + 5i$ Thus argument $\theta = \tan^{-1} \frac{y}{x}$ $= \tan^{-1} \frac{5}{12}$ $= 22.620°$ Modulus $r = \sqrt{(x^2 + y^2)}$ $= \sqrt{(12^2 + 5^2)}$ $= \sqrt{144 + 25}$ $= \sqrt{169}$ $= 13$	b) $z = -4 + 5i$ $\beta = \tan^{-1} \frac{y}{x}$ $= \tan^{-1} \frac{5}{-4}$ $= -51.34°$ Thus argument $\theta = 180 - 51.34$ $= 128.66$ Modulus $r = \sqrt{(x^2 + y^2)}$ $= \sqrt{((-4)^2 + 5^2)}$ $= \sqrt{(16 + 25)}$ $= \sqrt{41}$
c) $z = -4 - 5i$ $\beta = \tan^{-1} \frac{y}{x}$ $= \tan^{-1} \frac{-5}{-4}$ $= 51.34°$ Thus argument $\theta = 180 + 51.34$ $= 231.34$ Modulus $r = \sqrt{(x^2 + y^2)}$ $= \sqrt{((-4)^2 + 5^2)}$ $= \sqrt{(16 + 25)}$ $= \sqrt{41}$	d) $z = 4 - 3i$ $\beta = \tan^{-1} \frac{y}{x}$ $= \tan^{-1} \frac{-3}{4}$ $= -36.87°$ Thus argument $\theta = 360 - 36.87$ $= 323.13$ Modulus $r = \sqrt{(x^2 + y^2)}$ $= \sqrt{(4^2) + (3^2)}$ $= \sqrt{16 + 9}$ $= \sqrt{25}$ $= 5$

FORM IN COMPLEX NUMBER

- Cartesian or rectangular form $z = a + bi$
- Polar form $z = r \angle \theta$
- Exponential form $z = re^{i\theta}$ (note : θ must be in radians)

  Convert degree to radians $\theta° \times \frac{\pi}{180}$
- Trigonometric form $z = r(\cos\theta + \sin\theta)$

Tip: Step by step

i. Plot graph to identify argument.
ii. Calculate argument **θ**
iii. Calculate modulus **r**

Example :

Convert to polar ,trigonometric and exponential form

a) $z = 2 + 5i$

argument $\theta = \tan^{-1} \frac{y}{x}$

$= \tan^{-1} \frac{5}{2}$

$= 68.198°$

Modulus $r = \sqrt{(x^2 + y^2)}$

$= \sqrt{(2^2 + 5^2)}$

$= \sqrt{(4 + 25)}$

$= \sqrt{29}$

1) Polar form $z = \sqrt{29} \angle 68.198°$

2) Trigonometric form $z = \sqrt{29}\,(\cos 68.198° + i \sin 68.198°)$

3) Exponential form

Convert degree to radians

$68.198° \times \frac{\pi}{180} = 1.190$

$Z = \sqrt{29}\, e^{i1.190}$

CONVERT TRIGONOMETRY FORM TO CARTESIAN

Step by step

1) Expand the trigonometry form.
2) Solve the equation

Example:-

$Z = \sqrt{29}\ (\cos 68.182 + i\ \sin\ 68.182)$

Expand the trigonometry form

$Z = \sqrt{29}\ \cos 68.182 + \sqrt{29}\ i\ \sin\ 68.182$

Solve this equation

$Z = 2 + 5i$

CONVERT EXPONENT TO CARTESIAN

Step by step

1) Convert radians to degree

$$\theta_{rad} \times \frac{180}{\pi}$$

2) Change to trigonometric form
3) Solve the equation

Example :

$Z = \sqrt{29}\, e^{i1.190}$

Step 1

Convert radians to degree

$$\theta_{rad} \times \frac{180}{\pi}$$

$$1.190 \times \frac{180}{\pi} = 68.182°$$

Step 2

Change to trigonometric form

$Z = \sqrt{29}\ (\cos 68.182 + i \sin 68.182)$

Step 3

Solve this equation

$Z = \sqrt{29} \cos 68.182 + \sqrt{29}\, i \sin 68.182$

Cartesian form

$Z = 2 + 5i$

CONVERT POLAR TO CARTESIAN

STEP BY STEP

1) Change to trigonometric form
2) Solve the equation

Example :-

Z = $\sqrt{29} \angle 68.182$

Step 1

Change to trigonometric form

Z = $\sqrt{29}$ (cos 68.182+ i sin 68.182)

Step 2

Solve this equation

Z = $\sqrt{29}$ cos 68.182 + $\sqrt{29}$ i sin 68.182

Cartesian form

Z = 2 + 5i

MULTIPLICATION POLAR FORM IN COMPLEX NUMBER

$Z_1 = r_1 \angle \theta_1$ and $Z_2 = r_2 \angle \theta_2$

$$Z_1 Z_2 = r_1 r_2 \angle (\theta_1 + \theta_2)$$

$$Z_1 \div Z_2 = r_1 / r_2 \angle (\theta_1 - \theta_2)$$

Example :

1) If value $Z_1 = 8 \angle 40°$ and $Z_2 = 20 \angle 30°$. Find value of

a) $Z_1Z_2 = (8)(20) \angle (40° + 30°)$
$= 160 \angle (70°)$

b) $Z_1 / Z_2 = 8 / 20 \angle (40° - 30°)$
$= 2/5 \angle (10°)$

1) If value $Z_1 = 20 (\cos 40° + i \sin 40°)$ and $Z_2 = 4 (\cos 50° + i \sin 50°)$.
Find value of

a) $Z_1Z_2 = (20)(4) (\cos(40° + 50°) + i \sin(40° + 50°))$
$= 80 (\cos 90° + i \sin 90°)$

b) $Z_1 / Z_2 = (20/4) (\cos(40° - 50°) + i \sin(40° - 50°))$
$= 5 (\cos -10° + i \sin -10°)$