

A graphic for the chapter title. It features a blue-outlined cloud shape on the left containing the word 'CHAPTER' in bold black uppercase letters above the number '5' in bold black. To the right of the cloud is a horizontal rectangle with a black border containing the words 'VECTOR & SCALAR' in blue uppercase letters.

CHAPTER 5 VECTOR & SCALAR

5.1 Define vector

5.1.1 State the basic vector definition

- a. Vector notation
- b. Vector representation
- c. Equality of vectors
- d. Negative vector

5.1.2 Calculate:

- a. Unit vector
- b. Magnitude vector

5.2 Understand the operations of vectors

5.2.1 Demonstrate addition and subtraction of vectors using Parallelogram method.

5.2.2 Demonstrate addition and subtraction of vectors using triangle Construction method.

5.3 Apply scalar (dot) product of two vectors

5.3.1 Define scalar product

5.3.2 State properties of scalar product

5.3.3 Calculate the scalar product

5.4 Apply vector (cross) product of two vectors

5.4.1 Define vector product

5.4.2 State properties of vector product

5.4.3 Calculate the vector product.

5.5 Understand area of parallelogram

5.5.1 Calculate the area of parallelogram by using vector.

Course Learning outcomes (CLO) of this course :

Upon completion of this course, students should be able to:

1. Identify mathematical methods in solving the mathematical problems. (C2, LD1)
2. solve the mathematical problems by using appropriate techniques and solutions. (C3, LD1)
3. practice mathematical knowledge and skills in different mathematics problem. (C3, LD1)

5.1: Define Vector

Vector

Definition of Vector:

A quantity possessing both magnitude and direction, represented by an arrow the direction of which indicates the direction of the quantity and the length of which is proportional to the magnitude. Example of vectors are force, displacement, velocity and etc.

Scalar

Definition of Scalar:

A quantity that has a magnitude or length describable by a real number and no direction. Examples of scalars are time, volume, speed, temperature and etc.

5.1.1

a. Vector Notation

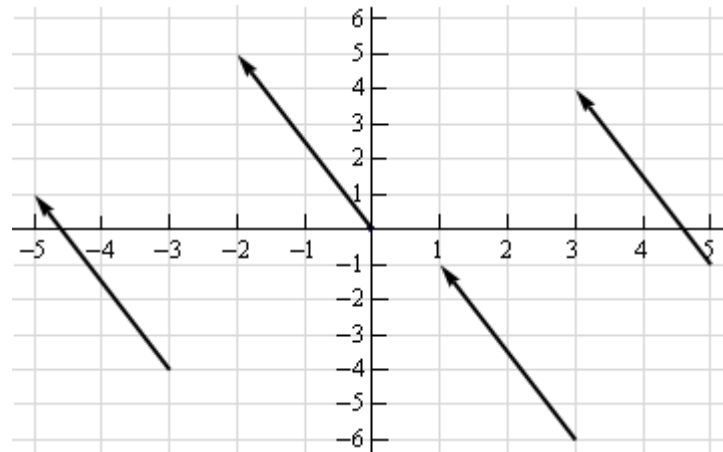
We will use a **bold capital letter** to name vectors. For example, a force vector could be written as **F**. Alternatively, some vectors can be written using an arrow above the vector name, like \vec{F}

A vector is drawn using an **arrow**. The **length** of the arrow indicates the **magnitude** of the vector. The **direction** of the vector is represented by the direction of the arrow.

b. Vector Representation

In a graphical sense vectors are represented by directed line segments. The length of the line segment is the magnitude of the vector and the direction of the line segment is the direction of the vector. However, because vectors don't impart any information about where the quantity is applied any directed line segment with the same length and direction will represent the same vector.

Consider the sketch below.



Each of the directed line segments in the sketch represents the same vector. In each case the vector starts at a specific point then moves 2 units to the left and 5 units up. The notation that we'll use for this vector is,

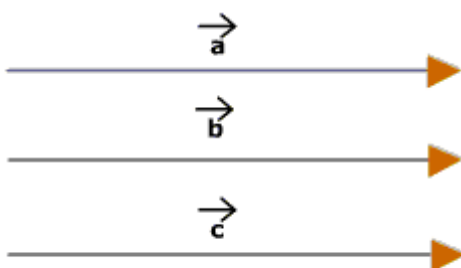
$$\vec{v} = \langle -2, 5 \rangle$$

and each of the directed line segments in the sketch are called **representations** of the vector.

c. Equality of vectors

Two vectors are said to be equal if they have the same magnitude and direction. In the figure below,

three equal vectors $\vec{a}, \vec{b}, \vec{c}$ have been represented. The equality of vectors is expressed in the following way $\vec{a} = \vec{b} = \vec{c}$



Equal vectors

Since the three vectors have the same direction,

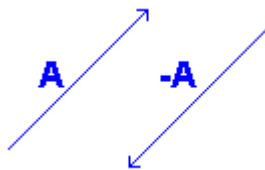
$$\hat{a} = \hat{b} = \hat{c}$$

Moreover, the three vectors are also equal in magnitude.

$$\therefore \left| \vec{a} \right| = \left| \vec{b} \right| = \left| \vec{c} \right|$$

d. Negative Vectors

A vector having the same magnitude but opposite direction to a vector **A**, is **-A**.



5.1.2 Calculate

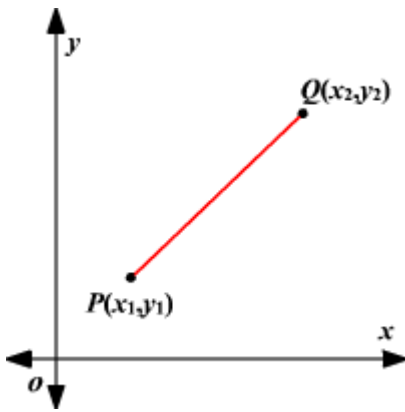
a. Unit Vector

A unit vector is a vector of unit length. A unit vector is often denoted by a lowercase letter with a \hat{u} . The formula for unit vector as below $\hat{u} = \frac{V}{|V|}$

b. Magnitude Vector

The magnitude of a vector \overrightarrow{PQ} is the distance between the initial point P and the end point Q . In symbols the magnitude of \overrightarrow{PQ} is written as $|\overrightarrow{PQ}|$. The magnitude of a unit vector is 1.

If the coordinates of the initial point and the end point of a vector is given, the Distance Formula can be used to find its magnitude.



$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 5.1

- Find the magnitude of the vector \overrightarrow{PQ} whose initial point P is at (1, 1) and end point is at Q is at (5, 3).

Substitute the values of x_1 , y_1 , x_2 , and y_2 .

$$\begin{aligned} |\overrightarrow{PQ}| &= \sqrt{(5-1)^2 + (3-1)^2} \\ &= \sqrt{4^2 + 2^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \\ &\approx 4.5 \end{aligned}$$

The magnitude of \overrightarrow{PQ} is about 4.5.

- A unit vector is a vector which has a magnitude of 1.

For example, the vector $\mathbf{v} = (1, 3)$ is not a unit vector because

$$\|\mathbf{v}\| = \sqrt{1^2 + 3^2} = \sqrt{10} \neq 1$$

The notation $\|\mathbf{v}\|$ represents the magnitude of vector \mathbf{v} .

But the vector $\mathbf{w} = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)$ is a unit vector because

$$\|\mathbf{w}\| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1$$

3. Find unit vector for $\overrightarrow{OP} = 4i - 2j + 3k$

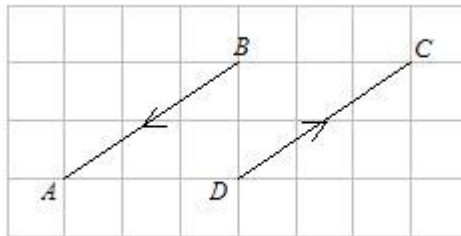
$$\hat{v} = \frac{\overrightarrow{OP}}{|\overrightarrow{OP}|}$$

$$|\overrightarrow{OP}| = \sqrt{4^2 + (-2)^2 + 3^2} = \sqrt{29}$$

$$\text{So, } \hat{v} = \frac{4i - 2j + 3k}{\sqrt{29}} = \frac{4}{\sqrt{29}}i - \frac{2}{\sqrt{29}}j + \frac{3}{\sqrt{29}}k$$

4. The negative sign reverses the direction of the vector.

A vector \overrightarrow{AB} is a negative vector of \overrightarrow{CD} , if \overrightarrow{AB} has the same magnitude as \overrightarrow{CD} but are opposite in directions.

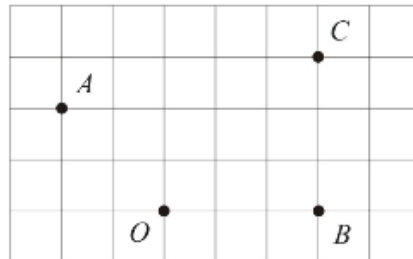


$$\overrightarrow{AB} = -\overrightarrow{CD} \text{ and } -\overrightarrow{AB} = \overrightarrow{CD}$$

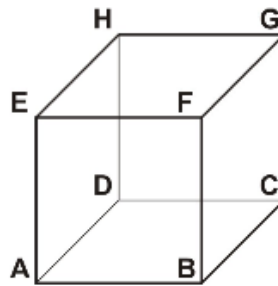
\overrightarrow{AB} and \overrightarrow{CD} are negative vectors of each other.

Exercise 5.1

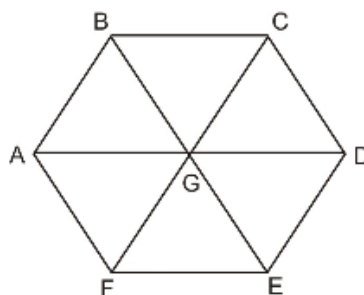
1. Draw the position vectors \vec{OA} , \vec{OB} , and \vec{OC}



2. Find three pairs of equivalent vectors in the diagram



3. Use the following diagram and identify three vectors parallel to \vec{AG}



4. It is given that $\vec{OM} = -5i + 2j$ and $\vec{ON} = i - 8j$. Find

- a) \vec{MN}
- b) the unit vector and direction of \vec{MN}

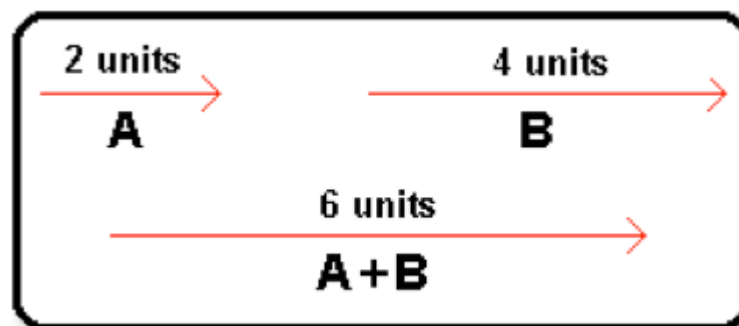
5. What is the magnitude of the vector $\mathbf{a} = (-2, 5)$?
6. Find the negative vector for $\vec{OP} = 5\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and then unit vector \vec{PO}

5.2 Understand the operations of vectors.

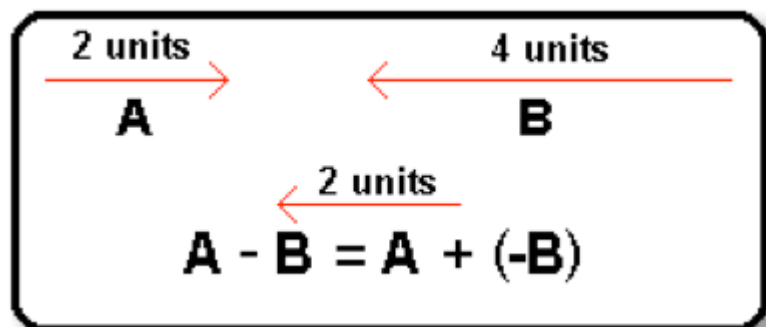
5.2.1 Demonstrate addition and subtraction of vectors using Parallelogram Method

Vector Addition and Subtraction

Two or more vectors may be added together to produce their addition. If two vectors have the same direction, their resultant has a magnitude equal to the sum of their magnitudes and will also have the same direction.



Similarly orientated vectors can be subtracted the same manner.



To add or subtract two vectors, add or subtract the corresponding components.

Let $\vec{a} = (a_1, a_2)$ and $\vec{b} = (b_1, b_2)$ be two vectors.

Then, the sum of \vec{a} and \vec{b} is the vector

$$\vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2)$$

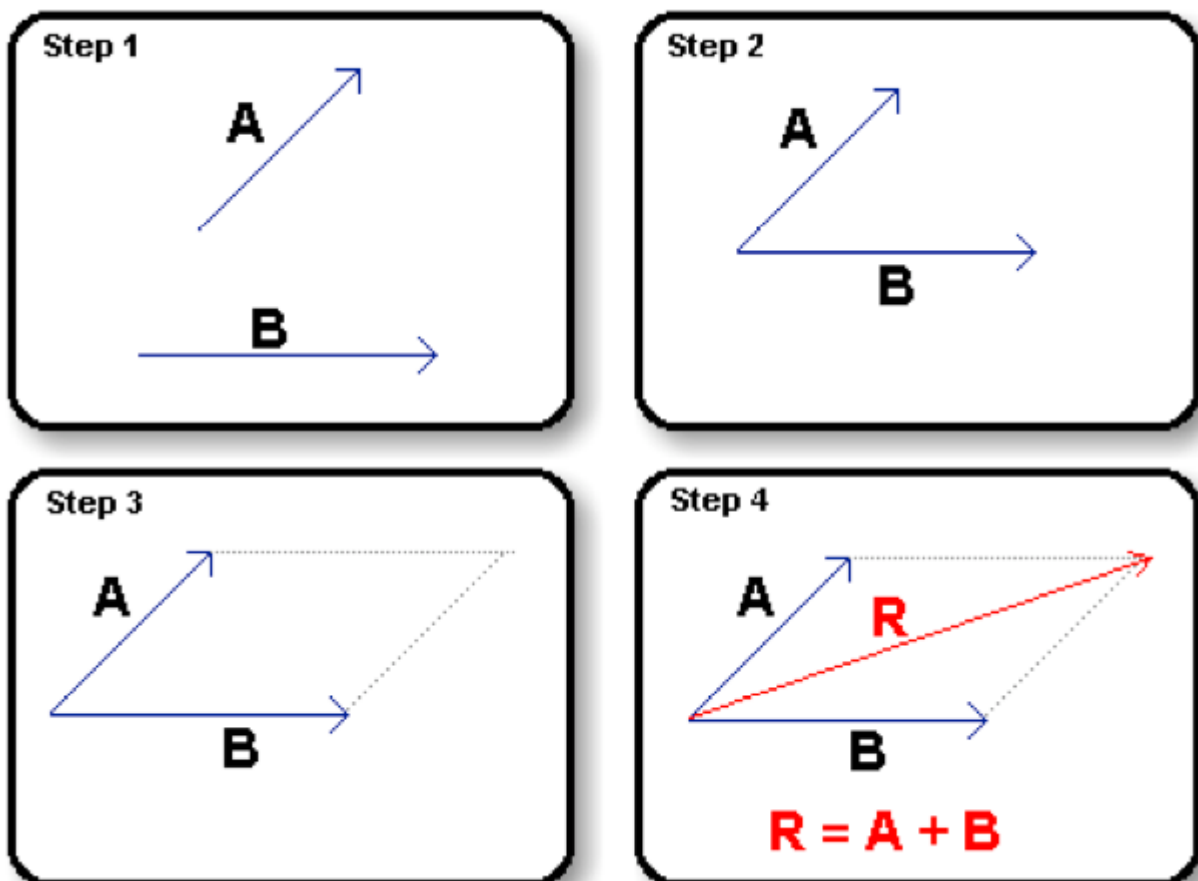
The difference of \vec{a} and \vec{b} is

$$\vec{a} - \vec{b}$$

$$= \vec{a} + (-\vec{b})$$

$$= (a_1 - b_1, a_2 - b_2)$$

The sum of two or more vectors is called the resultant. The resultant of two vectors can be found using the parallelogram method



Draw the vectors so that their initial points coincide. Then draw lines to form a complete parallelogram. The diagonal from the initial point to the opposite vertex of the parallelogram is the resultant.

Vector Addition:

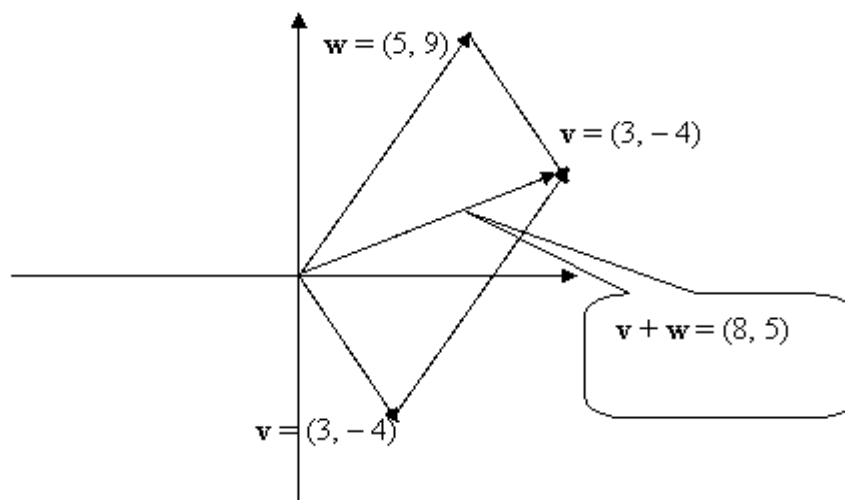
1. Place both vectors at the same initial point.
2. Complete the parallelogram. The resultant vector is the diagonal of the parallelogram.

Vector Subtraction:

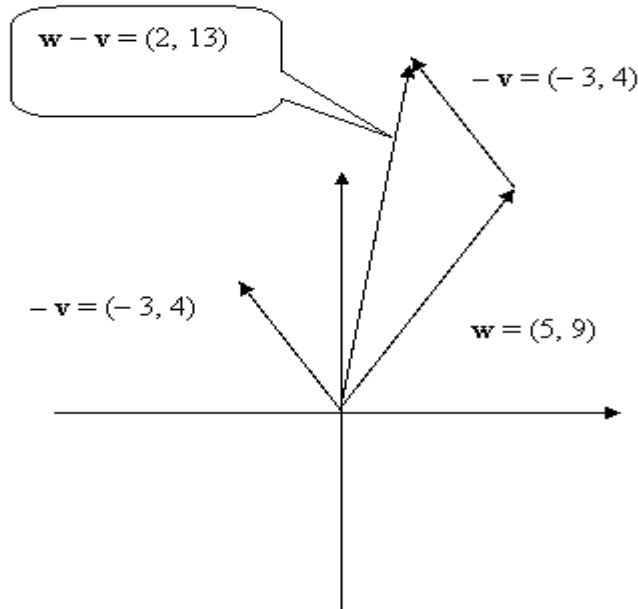
1. Complete the parallelogram
2. Draw the diagonals of the parallelogram from the initial point.

Example 5.2

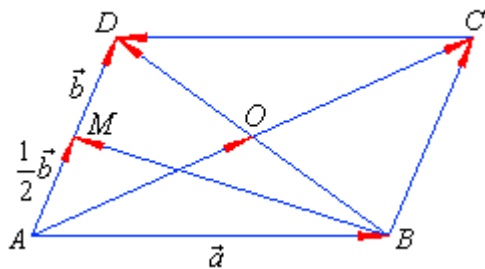
1. Notice that $\mathbf{v} + \mathbf{w}$ is the diagonal of a parallelogram formed by \mathbf{v} and \mathbf{w} . We translate the vector \mathbf{v} so that the tail (initial point) of \mathbf{v} is at the head (terminal point) of \mathbf{w} . We connect the tail of \mathbf{w} to the head of \mathbf{v} to diagram $\mathbf{v} + \mathbf{w}$ and it is the diagonal of a parallelogram formed by \mathbf{v} and \mathbf{w} . This is called “head to tail” addition of vectors.



2. To subtract $\mathbf{w} - \mathbf{v}$ we first diagram $-\mathbf{v} = (-3, 4)$ and translate $-\mathbf{v}$ so that the tail of \mathbf{v} connects with the head of \mathbf{w} . In other words, we add $\mathbf{w} + (-\mathbf{v})$. Drawing from the tail of \mathbf{w} to the head of the translated $-\mathbf{v}$ we construct $\mathbf{w} - \mathbf{v} = (2, 13)$. Notice this is in the exact opposite direction of $\mathbf{v} - \mathbf{w} = (-2, -13)$.



3. Given a parallelogram ABCD, the midpoint M of the side AD and the intersection point O of the diagonals. Express vectors \mathbf{AO} , \mathbf{BD} and \mathbf{BM} as the linear combination of vectors $\mathbf{a} = \mathbf{AB}$ and $\mathbf{b} = \mathbf{AD}$.



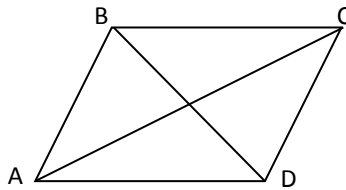
$$\text{As } \overrightarrow{AO} = \frac{1}{2} \overrightarrow{AC} \text{ and } \overrightarrow{AC} = \vec{a} + \vec{b}$$

$$\text{Then } \overrightarrow{AO} = \frac{1}{2} (\vec{a} + \vec{b}) = \frac{1}{2} \vec{a} + \frac{1}{2} \vec{b}$$

$$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = \vec{b} + (-\vec{a}) = \vec{b} - \vec{a}, \quad \overrightarrow{BM} = \frac{1}{2} \vec{b} - \vec{a}$$

Exercise 5.2.1

1. ABCD represents a parallelogram. State a single vector equal to each of the following.

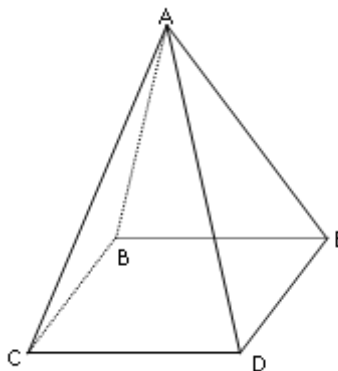


a. $\overrightarrow{AB} + \overrightarrow{AD}$

b. $\overrightarrow{BD} - \overrightarrow{BA}$

c. $\overrightarrow{AC} + \overrightarrow{BA} + \overrightarrow{CB}$

2. The diagram shows a square-based right pyramid. State any vectors equal to each of the following.



a. \overrightarrow{CB}

b. \overrightarrow{AB}

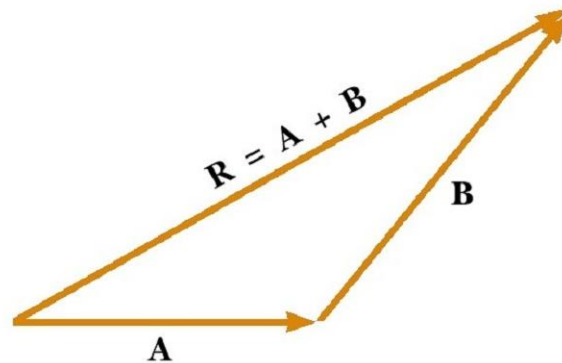
c. $\overrightarrow{AB} + \overrightarrow{BC}$

d. $\overrightarrow{AB} - \overrightarrow{AE}$

5.2.2 Demonstrate addition and subtraction of vectors using triangle construction method.

Adding Vectors

The rules for adding vectors are conveniently described by geometric methods. To add vector B to vector A, first draw vector A, with its magnitude represented by a convenient scale, on graph paper and then draw vector B to the same scale with its tail starting from the tip of A, as shown in figure below. The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is the vector drawn from the tail of A to the tip of B. This procedure is known as the triangle method of addition.



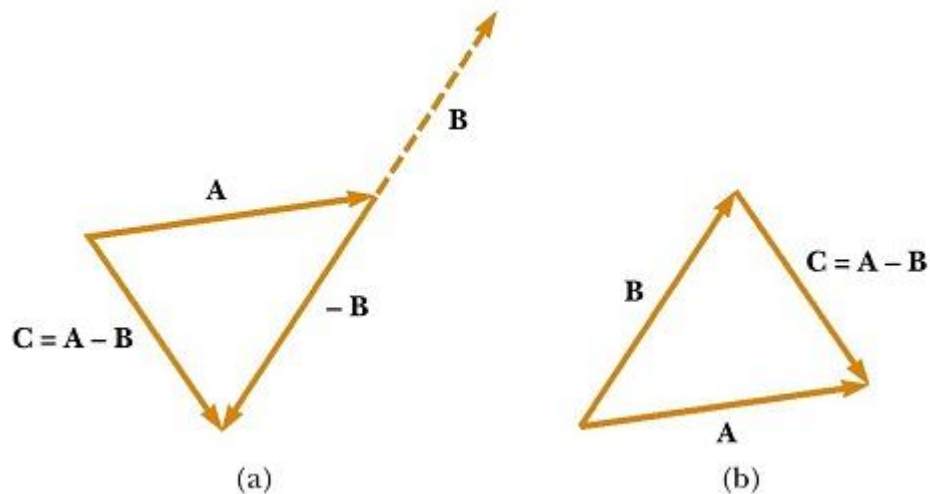
When vector B is added to vector A, the resultant R is the vector that runs from the tail of A to the tip of B.

Subtracting Vectors

The operation of vector subtraction makes use of the definition of the negative of a vector. We define the operation $\mathbf{A} - \mathbf{B}$ as vector $-\mathbf{B}$ added to vector A:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

Another way of looking at vector subtraction is to note that the difference $\mathbf{A} - \mathbf{B}$ between two vectors A and B is what you have to add to the second vector to obtain the first. In this case, the vector $\mathbf{A} - \mathbf{B}$ points from the tip of the second vector to the tip of the first, as the figure shows.



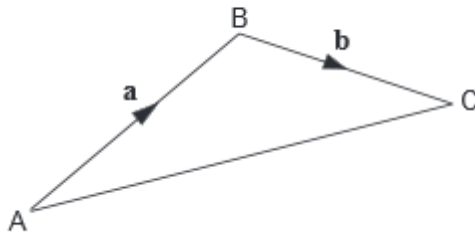
(a) This construction shows how to subtract vector B from vector A . The vector $-B$ is equal in magnitude to vector B and points in the opposite direction. To subtract B from A , apply the rule of vector addition to the combination of A and $-B$: Draw A along some convenient axis, place the tail of $-B$ at the tip of A , and C is the difference $A - B$. (b) A second way of looking at vector subtraction. The difference vector $C = A - B$ is the vector that we must add to B to obtain A .

Example 5.2.2

1. In the triangle ABC , \overrightarrow{AB} represents \mathbf{a} , \overrightarrow{BC} represents \mathbf{b} . If D is the midpoint of AB , express \overrightarrow{AC} , \overrightarrow{CA} and \overrightarrow{DC} terms of \mathbf{a} and \mathbf{b} .

Solution

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \mathbf{a} + \mathbf{b}$$



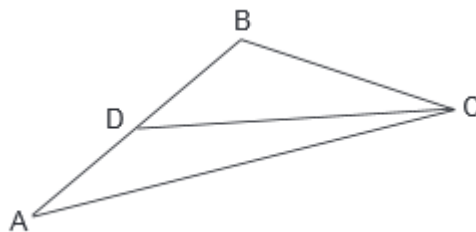
$$\overrightarrow{CA} = -\overrightarrow{AC} = -\mathbf{a} - \mathbf{b}$$

$$\overrightarrow{DC} = \overrightarrow{DA} + \overrightarrow{AC}$$

$$= -\overrightarrow{AD} + \overrightarrow{AC}$$

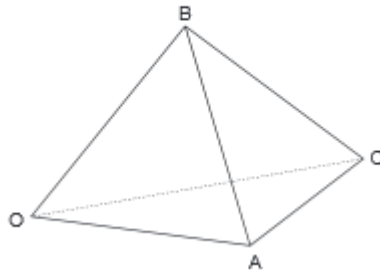
$$= -\frac{1}{2}\mathbf{a} + (\mathbf{a} + \mathbf{b}) \text{ since } D \text{ is the midpoint of } AB$$

$$= \frac{1}{2}\mathbf{a} + \mathbf{b}$$



2. $OABC$ is a tetrahedron with

$$\overrightarrow{OA} = \mathbf{a}, \overrightarrow{OB} = \mathbf{b} \text{ and } \overrightarrow{OC} = \mathbf{c}$$



Find \overrightarrow{AC} , \overrightarrow{AB} and \overrightarrow{CB} in terms of a, b and c and show that $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

Solution

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = b - a$$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = c - a$$

$$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = b - c$$

To show that $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$;

$$= \overrightarrow{AB} + [-(\overrightarrow{CB})] + [-(\overrightarrow{CA})]$$

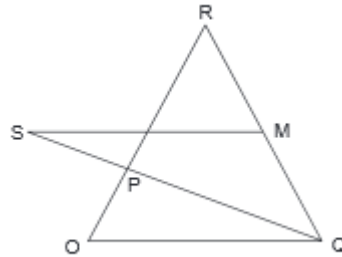
$$= (b - a) + [-(b - c)] + [-(c - a)]$$

$$= b - a - b + c - c + a$$

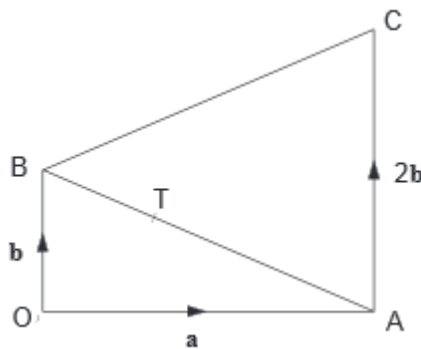
$$= 0 \text{ (proved)}$$

Exercise 5.2.2

1. $\overrightarrow{QP} = p, \overrightarrow{OR} = 3p, \overrightarrow{OQ} = q$. M is the midpoint of \overrightarrow{QR}



- Express \overrightarrow{OP} and \overrightarrow{RQ} in terms of p and q
 - Express \overrightarrow{MQ} in terms of p and q
 - If S lies on \overrightarrow{QP} produced so that $\overrightarrow{QS} = k \overrightarrow{QP}$, express \overrightarrow{MS} in terms of p, q and k
2. In the diagram, $TA = 2BT$, $\overrightarrow{OA} = a, \overrightarrow{OB} = b, \overrightarrow{AC} = 2b$



Find in terms of a and b

- \overrightarrow{AB}
- \overrightarrow{TB}
- \overrightarrow{BC}
- \overrightarrow{OT}
- \overrightarrow{TC}

5.3 Apply scalar (dot) product of two vectors

5.3.1 Define scalar product

The scalar product of two vectors a and b to be the magnitude of a multiplied by the magnitude of b multiplied by the cosine of the angle between a and b

We write this product as $a \bullet b$ and say this as “a dot b”.

Thus, $a \bullet b = |a||b|\cos\theta$ where this θ angle between a and b .

5.3.2 State properties of scalar product

1. $a \bullet b = b \bullet a$
2. $a \bullet a = a^2$
3. $a \bullet (b + c) = a \bullet b + a \bullet c$
4. $(a + b) \bullet c = a \bullet c + b \bullet c$
5. $a \bullet (\lambda b) = (\lambda a) \bullet b = \lambda(a \bullet b)$
6. For two perpendicular vectors, $a \bullet b = 0$

5.3.3 Calculate the scalar product

The dot or scalar product of vectors $A = a_1i + a_2j$ and $B = b_1i + b_2j$ can be written as

$$A \bullet B = a_1b_1 + a_2b_2$$

Two non-zero vectors are said to be **orthogonal** if their scalar product is zero. Obviously, two perpendicular vectors must be orthogonal since $\theta = \frac{\pi}{2}$, $\cos\theta = 0$, and so their scalar product is zero. For example, as i, j and k are mutually perpendicular, we have

$$i \cdot j = j \cdot k = k \cdot i = 0.$$

Also, as i, j and k are unit vectors, $i \cdot i = j \cdot j = k \cdot k = 1$.

The angle between two vectors

The angle between two nonzero vectors **A** and **B** is

$$\cos \theta = \frac{A \bullet B}{|A||B|}$$

Example 5.3

1. Vectors **A** and **B** are given by vector $A = 2i - 3j$ and vector $B = -4i + 2j$. Find the dot product $A \bullet B$ of the two vectors.

Solution:

$$A \bullet B = (2i - 3j) \bullet (-4i + 2j) = (2)(-4) + (-3)(2) = -14$$

2. Vectors **A** and **B** are given by $A = 2i - 3j + 7k$ and $B = -4i + 2j - 4k$. Find the dot product $A \bullet B$ of the two vectors.

Solution:

$$A \bullet B = (2i - 3j + 7k) \bullet (-4i + 2j - 4k) = (2)(-4) + (-3)(2) + (7)(-4) = -42$$

3. Determine the angle between $A = 2i + 2j$ and $B = 3j$

Solution.

$$|A| = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$|B| = \sqrt{3^2} = 3$$

$$A \bullet B = (2)(0) + (2)(3) = 6$$

$$\cos \theta = \frac{6}{(\sqrt{8})(3)}$$

$$\theta = 45^\circ$$

4. Determine if the following vectors are **orthogonal**: $A = 6i + 2j - k$ $B = 2i - 7j - 2k$

Solution:

The dot product is

$$A \cdot B = 6 \cdot 2 + 2 \cdot (-7) + (-1) \cdot (-2) = 12 - 14 + 2 = 0$$

So, the two vectors are orthogonal.

Exercise 5.3

1. Find the dot product for the following :

a) $A = 3i - 2j + 5k$ and $B = i - 7j + 4k$

b) $A = -5i - 3j - 8k$ and $B = 2i + 7k$

2 The position vectors of point A,B and C with respect of the origin are

$$OA = -6i - 2j + 4k, OB = -5i + 2j - 4k \text{ and } OC = -2i - 9j. \text{ Find}$$

a) AB

b) BC

c) $AB \bullet BC$

d) the angle between OB and OC

3. Find k so that:

$$\begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 6 \\ k \end{pmatrix} = 20$$

4. State whether the two vectors $i - 3j + 4k$ and $-i + j + k$ are orthogonal.

5.4 Apply vector (cross) product of two vectors

5.4.1 Define vector product

Vector product is a vector perpendicular to two given vectors, a and b , and having magnitude equal to the product of the magnitudes of the two given vectors multiplied by the sine of the angle between the two given vectors, usually represented by $a \times b$

$$\vec{a} \times \vec{b} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \hat{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \hat{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

5.4.2 State properties of vector product

1. $b \times a = -a \times b$
2. $a \times (b + c) = a \times b + a \times c$
3. For two parallel vectors, $a \times b = 0$
4. $i \times j = k, j \times k = i, k \times i = j$
5. $a \bullet (b \times c) = (a \times b) \bullet c$
6. $a \times (b \times c) = (c \bullet a)b - (b \bullet a)c$

Note:

- a) $a \times b \neq b \times a$
- b) $a \times (b \times c) \neq (a \times b) \times c$

Example 5.4

1. If $\vec{a} = 3i - 8j + 2k$ and $\vec{b} = 3i - 5j + 2k$, determine $a \times b$

Solution:

$$a \times b = \begin{vmatrix} i & j & k \\ 3 & -8 & 2 \\ 3 & -5 & 2 \end{vmatrix} = i \begin{vmatrix} -8 & 2 \\ -5 & 2 \end{vmatrix} - j \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} + k \begin{vmatrix} 3 & -8 \\ 3 & -5 \end{vmatrix}$$

$$\begin{aligned}
 &= i(-16 - (-10)) - j(6 - 6) + k(-15 - (-24)) \\
 &= i(-6) + k(9) \\
 &= -6i + 9k
 \end{aligned}$$

2. Let $a = -i + k$, $b = 2i + j - k$ and $c = i + 2j - 2k$. Find

a) $(a \cdot b)c$

b) $a \cdot (b \times c)$

Solution:

a) $a \cdot b = (-i + k) \cdot (2i + j - k) = (-1)(2) + (0)(1) + (1)(-1)$
 $= -3$

$$(a \cdot b)c = (-3)(i + 2j - 2k) = -3i - 6j + 6k$$

b) $(b \times c) = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 2 & -2 \end{vmatrix} = i \begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix} - j \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + k \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$

$$= i(-2 - (-2)) - j(-4 - (-1)) + k(4 - 1)$$

$$= i(0) - j(-3) + k(3)$$

$$= 3j + 3k$$

$$a \cdot (b \times c) = (-i + k) \cdot (3j + 3k)$$

$$= (-1)(0) + (0)(3) + (1)(3)$$

$$= 3$$

Exercise 5.4

1. Find a vector perpendicular to each of the following pairs of vectors:
 - a) $\underline{a} = (1, 4, 2)$, $\underline{b} = (3, 2, 7)$
 - b) $\underline{a} = (4, 1, -3)$, $\underline{b} = (2, -6, 4)$
 - c) $\underline{a} = (-2, 3, 1)$, $\underline{b} = (5, 2, 8)$

2. Determine the triple vector product of a with b and c in the cases where
 - a) $\bar{a} = i + j + 5k$, $\bar{b} = 3i - 5j + 2k$ and $\bar{c} = -4i + 3j$
 - b) $\bar{a} = -3i$, $\bar{b} = -6j - 4k$ and $\bar{c} = -4i + 3j$

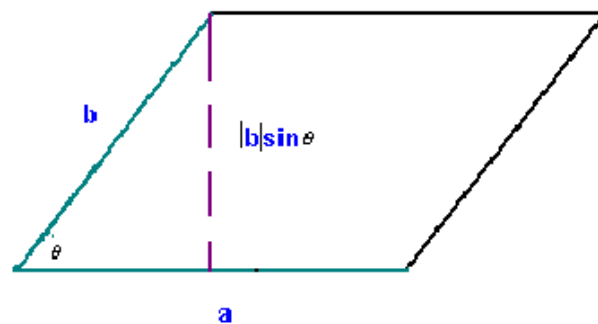
3. For the vectors $a = i + 2j + 3k$, $b = 2i + 3j + k$, $c = 7i + 2j + k$
 Find;
 - a) $a \cdot (b \times c)$
 - b) $a \times (b \times c)$

4. Find $a \times b$, $a \cdot (a \times b)$ and $b \cdot (a \times b)$ if $a = 3i + 2j - k$ and $b = i + 4j + k$

5.5 Understand area of parallelogram

5.5.1 : Calculate the area of parallelogram by using vector

In the previous section we combined two vectors to get a number, the dot product. In this section we see another way of combining two vectors, this time to get a vector, the cross product. Any two vectors in 3-space form a parallelogram. We define the cross product using this parallelogram

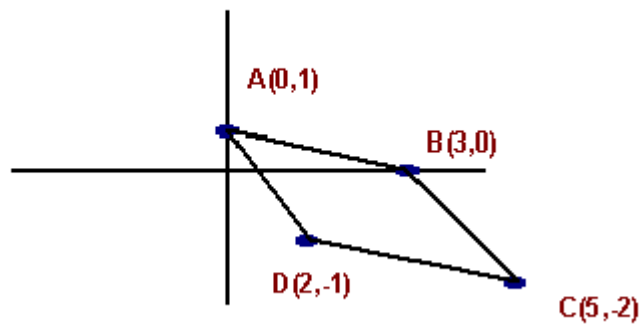


$$\text{Area } A = |a| (|b| \sin \theta) = |a \times b|$$

Example:

Find the area for the parallelogram with points A(0,1), B(3,0), C(5,-2), D(2,-1)

Solution:



$$a = |DC| = (5, -2) - (2, -1) = (3, -1)$$

$$b = |AD| = (0, 1) - (2, -1) = (-2, 2)$$

$$\text{Area } A = |a \times b| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 0 \\ -2 & 2 & 0 \end{vmatrix} \Rightarrow$$

$$\text{Area } A = \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 3 & 0 \\ -2 & 0 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 3 & -1 \\ -2 & 2 \end{vmatrix} \mathbf{k} \Rightarrow$$

$$\text{Area} = |0\mathbf{i} + 0\mathbf{j} + 4\mathbf{k}| = \sqrt{4^2} = 4$$

Exercise 5.5

- 1) Find the area of the parallelogram constructed by vectors AB and AC, with A(1,2), B(-3,4) and C(2,4)
- 2) If $\mathbf{v} = (1, 2, 3)$ and $\mathbf{u} = (-2, 6, 4)$, what is the area of the parallelogram formed by \mathbf{v} and \mathbf{u} ?